ABSTRACT

Title of thesis: IMPLICATIONS OF THE DICHOTOMY OF MODAL PARTICIPATION FACTORS FOR MONITORING AND CONTROL OF ELECTRIC POWER NETWORKS

Paul Kenton Tschirhart, Master of Science, 2013

Thesis directed by: Professor Eyad Abed
Department of Electrical Engineering

Steadily increasing demand for electricity has led to today's electric power networks often being stressed to their stability limits. Improved methods of stability monitoring and control placement are needed to manage the increased stress on power networks. Modal participation factors have been used for several decades in the analysis of modal behavior in power networks. Recently a dichotomy was discovered in modal participation, indicating that the participation of system states in system modes should be calculated differently from the participation of system modes in system states. This masters thesis numerically explores possible roles for these new participation factor definitions in power network monitoring and control. The results suggest that the mode in state participation factors are best employed in modal monitoring (especially in deciding which variables to monitor to best detect specific modes), while the state in mode participation factors are best suited to control applications (such as controller placement).

IMPLICATIONS OF THE DICHOTOMY OF MODAL PARTICIPATION FACTORS FOR MONITORING AND CONTROL OF ELECTRIC POWER NETWORKS

by

Paul Kenton Tschirhart

Thesis submitted to the Faculty of the Graduate School of the
University of Maryland, College Park in partial fulfillment
of the requirements for the degree of
Master of Science
2013

Advisory Committee:
Professor Eyad Abed, Chair
Professor Bruce Jacob, Advisor
Professor Alan Sussman

Acknowledgments

I would like to thank my principle advisors on this project, Professors Eyad Abed and Alan Sussman who have both been a source of constant encouragement, guidance and inspiration throughout the course of this work. Without their faith and inspiration this project would have definitely floundered early on. My work with them has also shaped much of what I now understand about the process of research and for that I will always be grateful.

I would also like to thank my PhD advisor Professor Bruce Jacob for supporting my completion of this project.

In addition, I would like to thank my friends and labmates who have provided good advice and valuable companionship during this project and over my time in graduate school in general.

Special thanks also goes to my family for their continual love and encouragement throughout the long and complicated journey that my graduate school education has been. Without their help listening and talking through the many problems that have come up, I probably would not have made it this far.

Finally, I would like to especially thank my fiance Tanya without whose love and support this process would have been much more difficult.

Table of Contents

List of Tables

List of Figures

Chapter 1: Introduction and Motivation

1.1 Power System Stability and Control

Since the year 2000 power blackouts have dramatically increased in number and severity worldwide as the complexity, interconnectivity, and load of power systems has increased [2,3]. To begin addressing this problem, improved methods are needed to accurately gauge voltage instability in the power system and effectively respond to disturbances with correctly placed control equipment. These methods would improve power system stability by providing early warning of potential problems and by reducing the severity of problems that do occur.

Power system stability generally refers to the ability of a power system to maintain a state of equilibrium under normal operating conditions and to return to that steady-state following a disturbance. There are three kinds of stability in power systems: frequency stability, rotor angle stability, and voltage stability. In this work we will focus on rotor angle stability and voltage stability. Rotor angle stability is traditionally the most studied form of power system stability. A decrease in rotor angle stability can result in the generators in the system losing synchronism which can cause widespread problems and lead to a blackout. Voltage stability is also important to understand because it can lead to voltage collapse which is a process that results in a blackout or abnormally low voltages in a significant portion of the power system. Therefore, recognizing instability and taking steps to remedy it before a minor disturbance can occur is an important component of power system stability protection. As a power system becomes more unstable, it becomes less able to recover from increasingly small disturbances. This increases the likelihood of voltage collapse or blackout in the system as the range of disturbances that can be tolerated shrinks. Furthermore, as power systems have become more interconnected, it has become more likely that a stability problem in one small part of the system could lead to large scale failures.

Once a disturbance has occurred, the ability to quickly return the system to a steady state is another important component of power system stability protection. This is accomplished by placing control equipment in key locations throughout the system that can react to disturbances and rapidly damp their effects. However, the increasing complexity of power systems has obscured the problem of control equipment placement. Further complicating matters, placing control equipment is an expensive process and often it is too expensive to simply place control equipment at every location in the system.

1.2 Participation Factors

In the past, modal participation analysis has been proposed as a means to extract some of the important information needed to accomplish the goals of better control placement and monitoring. For instance, modal participation analysis has been demonstrated to indicate the proper placement of control equipment [1, 4, 5]. Modal participation analysis produces two sets of scalar values called participation factors which describe the degree of participation of each system mode in each system state variable and vice versa. These values can be used to identify the critical locations of voltage instability in the system, exposing the optimal locations for control equipment. In addition, it has also been suggested that participation factors could be used as part of an online assessment of the systems proximity to voltage instability [6].

For many years it was thought that the measurement for the mode in state participation factors was identical to the measurement for the state in mode participation factors. Recently, however, it was determined that this was not the case. The original measurement accurately represented the mode in state participation factor but the state in mode participation factor needed to be calculated using a different method. As a result of this new dichotomy, it became unclear what information is captured by the different types of participation factors and what their applications might be with regard to power systems. Answering these questions is a vital step towards improving the methods for control placement and monitoring that rely on participation factors for key information.

1.3 Contributions of Thesis

This thesis proposes that state in mode participation factors are more relevant for control placement applications while mode in state participation factors are more relevant for monitoring applications. To test this hypothesis we perform two different sets of experiments using the Matlab toolkit PSAT [7] and the IEEE 9 and 14 bus test systems. One set of experiments determines whether the state in mode participation factor is particularly suited to control applications by evaluating its suggested placement of a power system stabilizer (PSS). Evaluating the effectiveness of mode in state participation factors for monitoring was a more complicated task that involved the development of a new Prony based voltage stability analysis method. By assessing the accuracy of this method when the mode in state participation factor suggested input signal was used, we were able to gauge how well suited mode in state participation factors are for monitoring applications. Additionally, though this new voltage stability analysis method was developed purely for use in the monitoring experiment, it could potentially be used to provide real-time voltage stability information in real world systems.

Mode in state participation factors provide a measure of the degree of participation of a particular mode in a particular state. This information could be used to obtain an accurate estimation of the value of a particular mode by analyzing the state that is most affected by that mode. Estimating the values of critical modes could provide valuable information about a power systems degree of voltage instability. Prior to voltage collapse, the real parts of certain critical system modes become smaller as the system nears the collapse. One way to identify the proximity of a system to voltage collapse would be to analyze these critical modes and determine how close to 0 their real components are. Prony analysis has been proposed as a way to estimate the values of a power systems modes [8–10]. However, Prony analysis works by decomposing a particular state variables signal and it is often not clear which state variables signal should be used. Additionally, choosing the wrong signal to decompose can lead to an inaccurate estimation. Mode in state participation factors indicate which signals should be used because they describe which signal is most affected by the mode in question. To evaluate this hypothesis, the load at a bus in the test systems is gradually increased, steadily pushing these systems closer and closer to voltage instability. A small disturbance is injected into the system at each step of the voltage increase to induce ringing in the state variable signals that the Prony method can then decompose. The modal content of each of the state variables signals is obtained with the Prony method and the estimated values of the critical mode are compared to its known values. These results show that the signal suggested by the mode in state participation factor consistently provides the most accurate estimation of the critical mode. Therefore, the hypothesis that mode in state participation factors provide useful information for monitoring application is experimentally confirmed by this work.

State in mode participation factors, on the other hand, are the measure of the degree of participation of a particular state in a particular mode. If a particular mode needs to be adjusted, it is logical that the state with the largest state in mode participation factor for that mode would have the greatest effect on it. Therefore, the state in mode participation factors should provide useful information for the placement of control equipment since this equipment should be placed where it will have the greatest effect. To test this hypothesis, a small fault was caused at a bus in the test systems and the response of each system was tuned to ensure that it reacted to this fault in an unstable fashion. Next, a power system stabilizer was added to different buses in the system and tuned to damp the response of the system as quickly and completely as possible. The effects the different power system stabilizer placements were then compared to the effect of the placement suggested by the mode in state and state in mode participation factors for the critical mode in each of the test systems. This comparison shows that the state in mode participation factors consistently suggest the best placement for the power system stabilizer, providing some confirmation of the hypothesis that state in mode participation factors provide relevant information for control placement.

The existing voltage stability analysis methods in the literature use participation factors to associate particular modes with particular locations and components in the system [6]. The voltage stability method proposed by this work, uses participation factors in a new way to inform the signal selection for Prony based modal analysis. By regularly using the Prony method to decompose key signals in the system, the state of critical modes in a power system could be calculated in near real time. This could provide a valuable on-line approximation of the systems proximity to voltage instability that is computationally simpler than other voltage stability assessment methods and potentially more accurate. To the best of our knowledge, no other work has been proposed using participation factors in this way.

The primary contributions of this thesis are as follows:

- A hypothesis regarding the implications for power systems applications of the recently discovered participation factor dichotomy is presented

- The suitability of mode in state participation factors for monitoring applications is determined with a Prony based voltage stability assessment experiment using the 9 bus and 14 bus IEEE test systems

- The applicability of state in mode participation factors for control placement applications is evaluated with a PSS placement experiment using two different test systems, the 9 bus IEEE test system and the 2 area test system

- A new Prony based voltage stability assessment method is proposed which utilizes mode in state participation factors to select the system signals to analyze

- A Matlab program has been developed for use with the PSAT toolkit which implements this new Prony based voltage stability assessment method

Chapter 2: Participation Factors

Participation factors are scalar values which quantify the degree of interaction between the state variables and modes (eigenvalues) of a linear time-invariant system. They were first introduced by Verghese, Perez-Arriaga and Schweppe in "Selective Modal Analysis with Applications to Electric Power Systems" in 1982 [11, 12]. Since then, participation factors have been used extensively in the area of electric power systems for stability analysis, equipment placement, and control design. Participation factors are useful for these applications because they can expose the relevant dynamics of complex power systems that would otherwise be difficult to analyze. It is also possible to determine these relationships between certain modes and certain state variables heuristically from a basic understanding of a particular system. However, participation factors provide a precise dimensionless measurement of those associations which can be used by other numerical techniques.

2.1 Calculation of Participation Factors

Linear time-invariant systems, such as power systems, have the general form

$$\dot{x}\,(t) = Ax(t) \tag{2.1}$$

where A is a real N x N matrix. In addition, for the sake of simplicity, we assume that A has a set of n eigenvalues that are distinct. From equation 1 it is clear that the values of each state variable in x can be affected by all of the eigenvalues (modes) of A. However, certain eigenvalues are more influential than others for each state variable. These associations between modes and state variables make up the dynamic pattern of behavior observable in the system described by the matrix A. When evaluating the behavior of that system is it often useful to be able to quantify and order the influence of the different modes on a particular state variable. In this way, the behaviors of interest can be studied by examining only their most associated state variables thereby reducing the complexity of the analysis. [11, 12]

Participation factors are computed from the left (row) and right (column) eigenvectors of the matrix A that are associated with each of As eigenvalues. The right eigenvector, $\mathrm{r_i}$, and the left eigenvector, l_{i}, corresponding to eigenvalue λ_{i} are computed for the matrix A as follows:

$$Ar_i = r_i\lambda_i \ , \ r_i \neq 0 \tag{2.2}$$

$$l_i^T A = \lambda_i l_i^T \ , \ l_i \neq 0 \tag{2.3}$$

The participation factor of the k^{th}state variable in the i^{th} mode is then computed by taking the product of the k^{th}entries of left and right eigenvectors corresponding to the i^{th}eigenvalue (mode):

$$p_{ki} = l_{ki}r_{ki} \tag{2.4}$$

One important property of the resulting values of this calculation is that they are dimensionless while the right and left eigenvectors are still dependent on units. The right eigenvector measures the activity of the kth state variable in the ith mode while the left eigenvector weighs the contribution of that state variable's activity to the mode. The right eigenvectors of the matrix A have been suggested as another way to measure the associations between different modes and states since the activity of a state for a particular mode is a good indicator of its effect on that mode. However, because the different entries in the right eigenvector can have different units, it is difficult to compare them. The dimensionless nature of participation factors solves this problem. Furthermore, the participation factor is not just measuring the activity of the state but also how much of that activity is actually affecting the mode because it is calculated with both the right and left eigenvectors. [11, 12]

2.2 Current Applications of Participation Factors

Currently, participation factors are primarily used to identify the critical locations in power systems for different operating points. To generate the participation factors, first a system matrix or power flow Jacobian is constructed for the power system based on measurements from the system and the state or power flow equations for that system [1, 3]. This Jacobian or system matrix is the equivalent of the A matrix discussed in the previous section and by solving for its right and left eigenvectors, the participation factors for the system can be determined. The participation factors that are derived from the power flow Jacobian will be limited to information relating to power flow such as the participation of buses in a particular mode. On the other hand, the full state matrix is often large and so calculating the participation factors for it generally is more computationally intensive. However, the participation factors calculated from the full state matrix include the participations of all of the system state variables in all of the system modes. Therefore, depending on the application that they are being used for, the power flow Jacobian or state matrix may be more appropriate.

Knowing the critical location in a particular power system can be especially useful when trying to decide where to place control or remedial equipment in the system. Frequently this equipment is expensive and so simply placing it at all of the available locations is redundant and overly expensive. Therefore, placing only as much equipment as is needed, in the correct locations, is necessary to achieve the desired effect. Participation factors that are calculated from the state matrix can provide the information needed to accomplish this goal. In particular, the participation factors of generator state variables in critical modes have been shown to indicate the optimal placement for power system stabilizers [1, 13, 14].

Voltage stability analysis also utilizes participation factors to identify the critical areas in the power system that are contributing the most to the voltage instability. The participation factors used for voltage stability analysis are usually calculated from the power flow Jacobian. This is because voltage instability is primarily the result of an inability to meet the demand for reactive power in the system which is a function of that system's power flow. By identifying which system buses have the largest participation factors for the critical mode during the instability;

power engineers can determine the best locations for control equipment.

2.3 New Dichotomy

The solution to equation (2.1) starting from some initial condition $x(0) = x^0$ can be written as

$$x(t) = e^{At}x^0 \tag{2.5}$$

If the eigenvalues of A are distinct, as we originally assumed, then A becomes similar to a diagonal matrix and equation (2.5) can be rewritten for $x_k(t)$ as

$$x_k(t) = \sum_{i=1}^{n} (l^i x^0) e^{\lambda_i t} r_k^i \tag{2.6}$$

In the original formulation of the participation factor for the ithmode in the kthstate, the authors chose the initial condition to be e^k which resulted in the original compact formula after some simplification

$$\begin{aligned} x_k(t) &= \sum_{i=1}^{n} (l_k^i r_k^i) e^{\lambda_i t} \\ &= \sum_{i=1}^{n} p_{ki} e^{\lambda_i t} \end{aligned} \tag{2.7}$$

To determine the inverse participation factor, the kthstate in the ith mode, the original system equation was first transformed with the similarity transformation to generate the following equation

$$z = V^{-1}x \tag{2.8}$$

The solution for this equation is

$$
\begin{aligned}
z(t) &= z_i^0 e^{\lambda_i t} \\
&= l^i x^0 e^{\lambda_i t} \\
&= \left[\sum_{i=1}^{n} l_k^i x_k^0 \right] e^{\lambda_i t}
\end{aligned}
\tag{2.9}
$$

Here again the selection of the initial condition can allow us to simplify this equation and derive a compact formula for the state in mode participation factor. By choosing the initial condition $x^0 = r^i$ we can generate the same equation for the state in mode participation factors as we had for the mode in state participation factors. [15]

$$
\begin{aligned}
z(t) &= z_i^0 e^{\lambda_i t} \\
&= \left[\sum_{i=1}^{n} l_k^i r_k^i \right] e^{\lambda_i t} \\
&= \left[\sum_{i=1}^{n} p_{ki} \right] e^{\lambda_i t}
\end{aligned}
\tag{2.10}
$$

Therefore, given the initial conditions that were selected by the authors of the first paper on participation factors the formula for the mode in state participation factor and the formula for the state in mode participation factor were the same. However, for systems that are operating near equilibrium, the initial condition of the system can be thought of as an uncertain vector near the system's equilibrium point. This uncertain set of initial conditions changes the definition of the mode in state participation factor to the following mathematical expectation:

$$
\begin{aligned}
p_{ki} &= E\left\{\frac{\left(l^i x^0\right) r_k^i}{x_k^0}\right\} \\
&= E\left\{\sum_{j=1}^{n}\frac{\left(l_j^i x_j^0\right) r_k^i}{x_k^0}\right\} \\
&= E\left\{\frac{\left(l_k^i x_k^0\right) r_k^i}{x_k^0}\right\} + E\left\{\sum_{j=1,\ j\neq k}^{n}\frac{\left(l_j^i x_j^0\right) r_k^i}{x_k^0}\right\} \\
&= l_k^i r_k^i + \sum_{j=1,\ j\neq k}^{n} l_j^i r_k^i E\left\{\frac{x_j^0}{x_k^0}\right\} \qquad (2.11)
\end{aligned}
$$

This version of the equation for the mode in state participation factors reduces to the original formulation if the components of the initial condition vector x^0 are independent and have zero mean. Therefore, the original formulation of the mode in state participation factors remains valid even with an uncertain initial condition. [15]

However, the same cannot be said for the state in mode participation factor. With an uncertain set of initial conditions the definition of the state in mode participation factor also becomes a mathematical expectation with the form:

$$
\begin{aligned}
\pi_{ki} &= E\left\{\frac{l_k^i x_k^0}{z_i^0}\right\} \\
&= E\left\{\frac{l_k^i \sum_{j=1}^{n} r_k^j z_j^0}{z_i^0}\right\} \\
&= E\left\{\frac{l_k^i r_k^i z_i^0}{z_i^0}\right\} + \sum_{j=1,\ j\neq i}^{n} l_k^i r_k^j E\left\{\frac{z_j^0}{z_i^0}\right\} \\
&= l_k^i r_k^i + \sum_{j=1,\ j\neq i}^{n} l_k^i r_k^j E\left\{\frac{z_j^0}{z_i^0}\right\} \qquad (2.12)
\end{aligned}
$$

The remaining expectation term in the equation can then be further reduced to yield the following expression for state in mode participation factors. [15]

$$\pi_{ki} = l_k^i r_k^i + \sum_{j=1,\ j\neq i}^{n} l_k^i r_k^j \frac{l^j (l^i)^T}{l^i (l^i)^T} \tag{2.13}$$

Unlike the mode in state participation factors which could be further reduced to the original participation factor definition by making some commonly valid assumptions, this expression for the state in mode participation values cannot be reduced to that original definition. Therefore, a dichotomy exists with regard to the two kinds of participation factors. They are not identical as was previously thought. [15]

This led to the motivating question of this thesis: if the two kinds of participation factors are not equivalent, what does this say about their potential applications in electric power systems? Originally, the participation factors could be seen as identifying critical locations in the system. The same location was the key to both proper placement of control equipment such as power system stabilizers and to proper understanding of instabilities. The new dichotomy, however, suggests that optimum locations in the system for these two kinds of applications might be different. Therefore, instead of having a single location that is critical for both control and monitoring, some locations in the system could be optimal for monitoring while others could be optimal for control.

Chapter 3: Voltage Stability Analysis

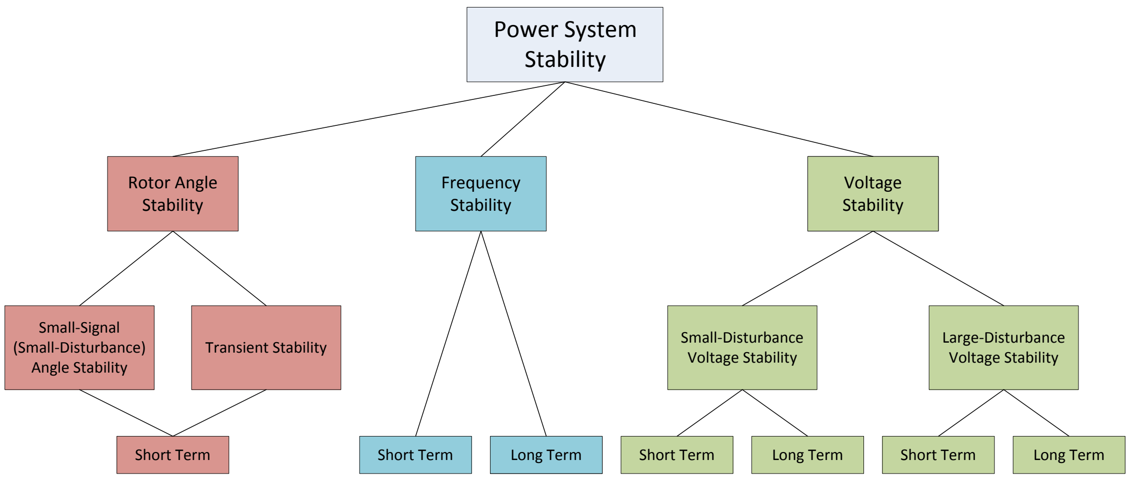


Figure 3.1: Classification of the different forms of power system stability.

Power system stability is the ability of a power system to maintain a steady-state equilibrium under normal operating conditions and to return to an acceptable equilibrium following a disturbance. Figure 3.1 illustrates the three categories of power system stability: frequency stability, rotor angle stability and voltage stability. The frequency stability of an electric power system is its ability to maintain a steady frequency throughout the system after a disturbance that results in an imbalance between generation and load. Rotor angle stability indicates how well the interconnected synchronous machines of a power system can remain in synchronism after a disturbance. Rotor angle stability is further divided into small-signal (small-disturbance) stability and transient stability. In this context, small-signal stability is the systems ability to deal with small variations in load and generation while transient stability is the systems ability to deal with large disturbances. Finally, voltage stability refers to the power systems ability to maintain a steady voltage at all buses in the system following a disturbance. Like rotor angle stability, it is also further divided into small-disturbance stability and large-disturbance stability. However, while rotor angle stability is a measure of generator stability, voltage stability is a measure of load stability. [16]

In the monitoring chapters of this thesis we will focus on voltage stability. Historically, rotor angle transient stability has been the primary stability problem for most systems and was subsequently the focus of much of the work in the area of power system stability [17] However, the other types of instability have become more significant as power systems have continued to develop in terms of the growth of interconnections, increased operation in highly stressed conditions and the adoption of new technologies. Electric utilities have been forced to maximize the utilization of their transmission capabilities as a result of economic and environmental pressures. As a result, voltage stability has surpassed rotor angle transient stability to become the limiting factor of many systems. The importance of voltage stability in modern power networks has been demonstrated by its role in recent major blackouts in the last few decades. [18, 19]

There are two aspects to voltage stability analysis: determining the proximity of the system to instability and determining the mechanism of instability that has occurred [6]. The proximity aspect of voltage stability can be used to predict an impending instability, giving operators time to address the situation before a serious problem can occur. On the other hand, understanding the typical sources of instability in a system can help determine future modifications or operating strategies that can minimize or prevent voltage instability. The work of this thesis focuses primarily on an improved method for determining the systems proximity to voltage instability. However, the new participation factor definitions could also be used to improve methods for determining the mechanisms of voltage instability.

3.1 Static Methods

Both static and dynamic methods can be used to study voltage stability. The static methods utilize data from a snapshot of the system at a point in time and the power flow equations to determine the behavior of the system at that point. Dynamic methods perform time domain simulations of the system utilizing both differential and algebraic equations. This provides a detailed view of the sequence of the events that led to the instability but takes considerably longer than the static methods.

Many different methods for static voltage stability analysis are presented in the literature including V-P curves, Q-V curves, the minimum singular value stability index and modal analysis. V-P curves were first proposed by Balamourougan in [20]. To create the V-P curve for a bus in the system the real power load at that bus in gradually increased until the voltage drops below some acceptable level. This provides a maximum real power loading value for that bus beyond which the system will be voltage unstable. Solving the power flow equations to generate each point on the curve is relatively easy with computers. However, to gain a system-wide perspective of the voltage stability, V-P curves have to be generated for many buses. In large complex systems this can become a confusing and time consuming process. Another downside of V-P curves is that they can only provide information about the maximum load at buses in the system. They cannot provide information about the mechanism of instability.

Q-V curves are similar to V-P curves but are concerned with the reactive power that needs to be injected at a bus to maintain a voltage level [21]. They are generated by gradually increasing the voltage level and measuring the amount of reactive power that is injected at the bus to maintain that voltage. Like V-P curves, Q-V curves provide maximum loading value for a bus but they are a measure of the maximum reactive power load instead of maximum real power load. Q-V curves have the same downsides as V-P curves. The curves must be generated for many buses in order to gain a system-wide perspective of the stability situation. That process can be time consuming and confusing in large systems. Also, Q-V curves cannot provide any information about the causes of instability.

A minimum singular value stability index is generated by performing singular value decomposition on the power flow Jacobian matrix [22]. The smallest singular value that results from this operation can be used to provide an approximate measure of the proximity of the system to voltage instability. When the smallest singular value is equal to zero then the system has bifurcated and become voltage instable. This method, therefore, provides a relatively straightforward and fast way to determine the systems proximity to voltage instability. However, the approach of the system from a stable operating point to bifurcation is not linear so the proximity measurement is not absolute. Also, like V-P curves and Q-V curves, this method cannot provide any information about the causes of the instability. [3]

Unlike the other static methods discussed, modal analysis is able to provide information about the relative stability of the system as well as the information about the mechanisms of instability within the system. Proximity information comes from the eigenvalues of the power flow Jacobian or the system state matrix. As the system becomes more unstable, the magnitude of at least one of its eigenvalues will become smaller. This eigenvalue or eigenvalues which become smaller are the critical eigenvalues (modes) of the system. When the magnitude of the eigenvalue becomes zero then the system has bifurcated and has become voltage unstable. In this way modal analysis provides an approximate measure of proximity to voltage instability similar to measure provided by the singular value method. The modal analysis proximity measurement is also not absolute as a result of the nonlinear path to bifurcation. Information about the mechanism of instability is derived from the participation factors of the power flow Jacobian or the system state matrix. The buses, branches and generators which have large participation factors for the critical modes of the system are the areas that are most prone to voltage instability. [3, 6]

3.2 Dynamic Methods

Dynamic voltage analysis methods have been proposed for use in conjunction with static methods to provide more detail for certain important disturbance scenarios (contingencies) [18]. In this scheme static methods are used to determine the voltage stability margins for all of the different contingency cases of interest. Then more detailed time domain simulations are used to establish the voltage stability margin for a few select contingency cases that are especially critical. In this case the dynamic analysis consists of performing the time domain simulation for each contingency and determining if the system reaches an acceptable equilibrium point after the disturbance. The time domain simulation of a voltage inst-able system will not reach equilibrium after the disturbance and will have bus voltages that continue to decrease. These time domain simulations are accurate but time consuming. Therefore, by only using time domain simulations to analyze the most important contingencies the overall computation time is minimized while improved accuracy is provided for key scenarios.

All of the methods proposed thus far, both static and dynamic, rely on a system model simulation to generate their measurements. Therefore, the accuracy of the measurement is limited by the complexity of the simulation. It would be ideal to be able to get a measurement of the systems proximity to voltage instability directly from the system. One way to accomplish this is to utilize Prony analysis to extract the modal content of system from the response signals of state variables [9, 23]. The modal content obtained from Prony analysis can then be used to determine the magnitude of the smallest eigenvalue in the system, establishing the proximity to bifurcation and voltage instability. This measurement is similar to the measurements produced by the smallest singular values and static modal analysis methods. However, choosing incorrect settings for the Prony analysis or performing the analysis on the wrong state variable response can greatly reduce the accuracy of the resulting measurements. In large or complex systems, determining these settings and the correct state variable to analyze can be difficult and confusing. As a result Prony analysis has not seen across the board successful application in the literature [24].

3.3 Role of Participation Factors

Participation factors are currently used in the area of voltage stability analysis primarily to identify the critical areas in systems. This information can then be used to determine the optimal siting of equipment to prevent future voltage instability. However, participation factors could be used to extract more detailed information about the system. In most approaches the application of participation factors is limited because they are derived from the reduced power flow Jacobian. As a result there are only participation factors for buses, branches and generators for the critical modes. If the full system state matrix were used to generate the participation factors then participation factors could be calculated for all of the state variables in the system. This would be more computationally intensive but would provide more information about the dynamics of the system.

In particular, this additional information about the system could be used to determine which state variables to analyze with Prony analysis. The state variables with the largest mode in state participation factors for the critical modes are the state variables which are most affected by those modes. Therefore, the responses of those state variables are logically best suited to Prony analysis. Analyzing these responses should result in more accurate measurements from the Prony analysis and a more accurate estimate of the systems proximity to voltage instability. The work of this thesis investigates that hypothesis and provides some supporting numerical evidence.

Chapter 4: Prony Analysis

Prony analysis was developed in 1795 by Gaspard Riche, Baron de Prony as a way to describe the expansion of various gases. The modern version of Prony's method is very different from the original approach proposed in 1795 due to evolutionary changes. [25] The original version performed an exact fit of an exponential curve that had p exponential terms to 2p data points. The more modern approach utilizes an estimation procedure, such as least squares, to approximately fit a curve with p exponentials to a data set that has N samples where N > 2p. This development and others which have helped to address Prony's inherent numerically ill-conditioned mathematics, combined with the power of the digital computer, have enabled the first practical applications of Prony nearly two centuries after its conception. [8, 25]

To understand how Prony analysis performs its curve fitting let us revisit the linear, time-invariant dynamic system that we discussed in Chapter 2:

$$\dot{x}\left(t\right) = Ax(t) \tag{4.1}$$

The solution to this system could be expressed as:

$$x(t) = \sum_{i=1}^{n} (l_i x_0) e^{\lambda_i t} r_i \tag{4.2}$$

where l_i, r_i, and $\lambda_i$represent the left eigenvectors, right eigenvectors and eigenvalues respectively of the matrix A from equation (4.1) and x_0 represents the initial state at time t_0. If we assume for simplicity that this system has just one output and it has the form:

$$y(t) = Cx(t) \tag{4.3}$$

then Prony analysis can estimate the exponential terms in equation (4.2) by fitting the function

$$\hat{y}(t) = \sum_{i=1}^{N} A_i e^{\sigma_i t} \cos(((2\pi) f_i t) + \phi_i) \tag{4.4}$$

to the observed records for the function $y(t)$. For applications like determining the modal content of power system response, the data set for $y(t)$ consists of N measurements which are taken at regular time interval of Δt. Given this regular set of data then the algorithm of determining the Prony solution is [9, 24]:

Step 0. Data Preprocessing Prony analysis relies on relative changes around some steady state position therefore any constant or steady state trends need to be removed to ensure an accurate estimation.

Step 1. Estimation Construction A discrete linear prediction model is generated with an estimation method, such as the least squares method or the Kalman filter method, which fits the data set. This linear prediction model will have a characteristic polynomial of the form:

$$y(N+k) = a_1 y(N+k-1) + \cdots + a_p y(k) \tag{4.5}$$

<u>*Step 2. Determine the Roots*</u> A root solving routine is used to find the roots of the characteristic polynomial from Step 1.

<u>*Step 3. Calculate Amplitude and Initial Phase*</u> The roots from Step 2 are used as the complex modal frequencies of the signal enabling the calculation of the amplitude and initial phase of each mode.

<u>*Step 4. S-Domain Translation*</u> All of the above steps are carried out in the z-domain and need to be translated into the s-domain to produce the damping and natural frequency of oscillation of the system modes.

[8, 24]

Step 1 of this algorithm begins with the following expression for the estimated function which is the exponential form of equation (4.4)

$$\hat{y}(k) = \sum_{i=1}^{N} B_i z_i^k \tag{4.6a}$$

where

$$z_i = e^{\lambda_i \Delta t} \tag{4.6b}$$

The goal of Prony analysis is to determine the values for B_i and z_i that will make $\hat{y}(k) = y(k)$ true for all k. One way to accomplish this is to first construct an equation that expands equation (4.6a) for each t_k

$$\begin{bmatrix} B_1 z_1^0 & \cdots & B_n z_n^0 \\ \vdots & \ddots & \vdots \\ B_1 z_1^{N-1} & \cdots & B_n z_n^{N-1} \end{bmatrix} = \begin{bmatrix} z_1^0 & \cdots & B_n z_n^0 \\ \vdots & \ddots & \vdots \\ B_1 z_1^{N-1} & \cdots & B_n z_n^{N-1} \end{bmatrix} \begin{bmatrix} B_1 z_1^{N-1} \\ \vdots \\ B_n z_n^{N-1} \end{bmatrix} = \begin{bmatrix} y(0) \\ \vdots \\ y(N-1) \end{bmatrix} \tag{4.7a}$$

$$ZB = Y \tag{4.7b}$$

The $z_i$values are necessarily the roots of an n-order polynomial equation with a set of unknown coefficients a_i so the following equation must be true

$$z^n - \left(a_1 z^{n-1} + a_2 z^{n-2} + \cdots + \ a_n z^0\right) = \ 0 \tag{4.8}$$

If we then construct the following matrix

$$\overline{A} = [-a_n \ -a_{n-1} \ldots \ -a_n \ 1 \ 0 \ldots 0] \tag{4.9}$$

Equation (4.7) can be rewritten as

$$\begin{aligned} \overline{A}Y &= y\left(n\right) - \left[a_1 y\left(n-1\right) + a_2 y\left(n-2\right) + \cdots + \ a_n y\left(0\right)\right] \\ &= \overline{A}ZB \\ &= B_1\left[z_1^n - \left(a_1 z_1^{n-1} + a_2 z_1^{n-2} + \cdots + \ a_n z_1^0\right)\right] + \ldots \\ &= 0 \end{aligned} \tag{4.10}$$

This is true because Z = 0 from equation (4.8). Therefore we can express $\overline{A}Y$ as the matrix equation

$$\begin{bmatrix} y(n-1) & y(n-2) & \dots & y(0) \\ y(n-0) & y(n-1) & \dots & y(1) \\ y(n+1) & y(n-0) & \dots & y(2) \\ \vdots & \vdots & \dots & \vdots \\ y(N-2) & y(N-3) & \dots & y(N-n-1) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} y(n+0) \\ y(n+1) \\ y(n+2) \\ \vdots \\ y(N-1) \end{bmatrix} \tag{4.11}$$

Solving this equation gives us values for a_n which can be used in equation (4.8). Equation (4.8) can then be evaluated to determine the root values for z_i and from the roots we can calculate the eigenvalues of the system using equation (4.6b). This process covers steps 1 and 2 of the Prony solution algorithm. To perform step 3 we solve equation (4.7) for B_i using the root values for z_i that we just calculated. Finally to translate the roots from the z-domain into the s-domain the following conversions are used provided that $z_i$is a complex conjugate pair

$$s_i = \alpha_i \pm j\beta_i \tag{4.12a}$$

where

$$\alpha_i = \frac{1}{\Delta \mathrm{t}} ln\,|z_i| \tag{4.12b}$$

$$\beta_i = \frac{1}{\Delta \mathrm{t}} \tan^{-1} \left\{ \frac{z_{li}}{z_{Ri}} \right\} \tag{4.12c}$$

$$z_i = z_{Ri} \pm j z_{li} \tag{4.12d}$$

These values can also then be used to determine the damping (δ_i) and natural frequency (ω_n)

$$\omega_n = \left(\alpha_i^2 + \beta_i^2\right)^{1/2} \tag{4.13}$$

$$\delta_i = \frac{-\alpha_i}{\omega_n} \tag{4.14}$$

That completes step 4 of the Prony solution algorithm. [8, 24]

Finally, the estimated signal $\hat{y}(t)$ will usually not fit $y(t)$ exactly. A signal-to-noise ratio is typically used to quantify the accuracy of the estimated signal. [8]

$$SNR = 20log \left\| y(k) - \hat{y}(k) \right\| / \left\| y(k) \right\| \tag{4.15}$$

In this equation the $\|.\|$ operator indicates a root-mean-squared norm. [8]

4.1 Current Applications

Prony analysis allows us to estimate the eigenvalues of a system based on a set of sampled data from a response curve of that system. In power systems this response is usually a ring-down response to some intentional minor disturbance in the system. The eigenvalues that Prony analysis identifies in its estimate of that response curve are themselves estimates of the characteristic eigenvalues of that system. Those eigenvalues can then be used to approximate the proximity to voltage instability of that system as was discussed in Chapter 3. Therefore, Prony analysis could be used to create an online voltage instability predictor that would warn operators of impending voltage stability problems [26]. Furthermore, the damping and frequency information produced by Prony analysis can also be used to tune the output of a power system stabilizer in the system [26].

To determine its effectiveness, Prony analysis has been applied to simulations and to real world power systems and has been shown to produce relatively accurate estimates of the modal content of those systems [9, 23]. However, these results rely on the proper selection of the modal order (n) and the sampling interval (Δt) for the analysis. This is often not a straightforward task [24]. A suboptimal selection for these values greatly decreases the accuracy of the resulting estimation and reduces the value of Prony analysis. So, while Prony is potentially very useful for power systems applications, it is often difficult to obtain satisfactory results using it.

Other oscillatory signal analysis techniques like Fourier analysis could also be used to analyze the ring-down response signals and determine their composition. However, Prony has the advantage that it best fits a reduced-order model to the high order system in both the time and frequency domains [27]. In addition, Prony also has the advantage of computing the damping coefficients separately from the frequency, phase, and amplitude of the signal [28]. Therefore, despite its difficulties, Prony analysis is the best choice for power systems applications. As a result, a great deal of work can be found in the literature developing power systems application for Prony analysis.

Chapter 5: Mode in State Participation Factors

Prony analysis is a potentially very useful tool for power system engineers. Using it to analyze the ring down that results from small intentional perturbations to the power system could provide operators with information about the modes of that system. This information could then be used to influence the outputs of control equipment or estimate the system's proximity to voltage collapse [3, 26]. One major advantage of Prony analysis is it uses the real-world responses from the system and is therefore potentially more accurate. Other methods that have been developed to extract the important modal information from the system, such as the modal analysis technique proposed in [3], rely on simulated models. As a result, these other methods are only as accurate as their simulations.

However, the accuracy of Prony analysis is dependent on the proper selection of sampling time interval and modal order [24]. This thesis demonstrates that the selection of the state variable response to analyze has a potentially large effect on the accuracy of the Prony. Some state variables appear to have responses that are better suited to Prony analysis than others. Furthermore, the state variables that are best suited to Prony analysis are not always the intuitive choices. Therefore, in order to fully utilize Prony some method is needed to direct the choice of which state variable response to analyze.

Mode in state participation factors are a logical choice as a source of information about which state variables might be best suited for Prony. They represent the magnitude of effect that each mode of the system has on each of the state variables. In other words, the state variable that has the highest mode in state participation factor for a particular mode is the state variable that is most influenced by that mode. Therefore, the mode should be clearly present in the state variable's response of the mode-state pair with the highest mode in state participation factor. Performing Prony analysis on that response should then yield an accurate approximation of the current values of the corresponding mode. Furthermore, the state in mode participation factors should not be as useful as the mode in state participation factors because they do not contain the same information.

This thesis presents numerical results from time domain simulations of two different test systems that support the hypothesis that mode in state participation factors in particular indicate prime candidates for Prony analysis from among a system's state variable responses.

5.1 Proposed Stability Analysis Technique

To evaluate the effectiveness of mode in state participation factors for selecting candidates for Prony analysis a point of comparison is required. The results of a modal analysis method similar to the one proposed by [3] are used for this purpose. Since the Prony analysis is being performed on responses from the simulated system

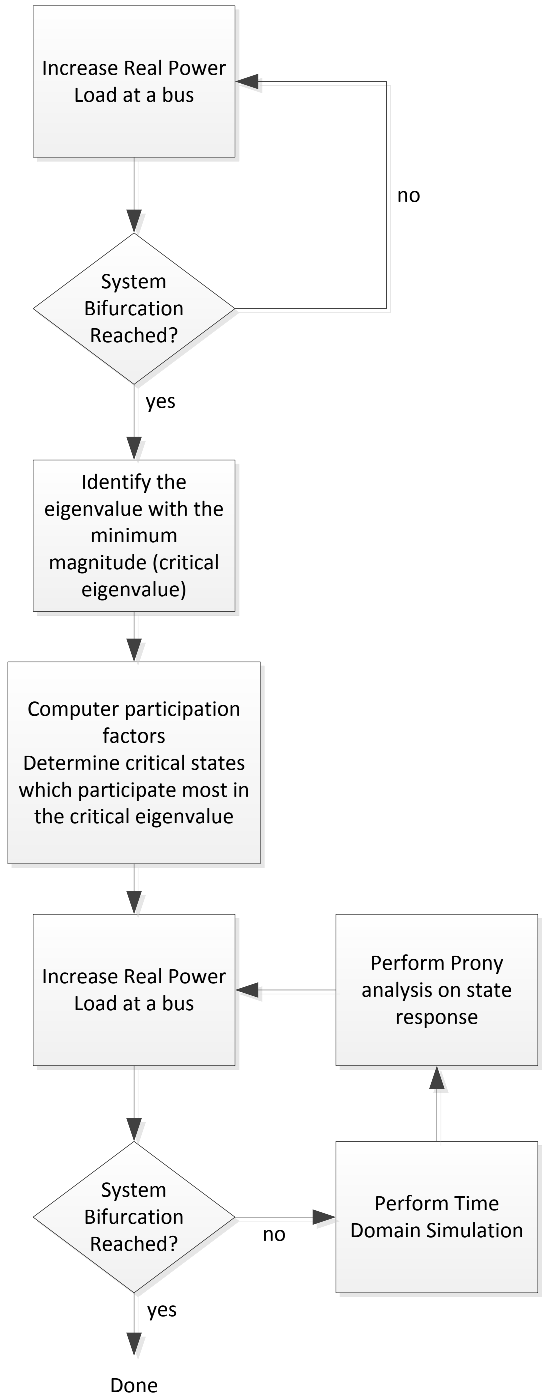


Figure 5.1: Participation Factor Directed Prony Modal Stability Analysis Algorithm.

in this case, the modal analysis results represent the actual values of the system modes. By comparing the Prony estimated values of the modes to the actual values of those modes, we can establish the relative accuracy of the Prony analysis for each state variable response. If the state variables suggested by the mode in state participation factors consistently provide accurate estimates of the critical modes while other state variables do not then it can be said that the mode in state participation factors are providing useful information for Prony analysis.

The comparison process used by this thesis is presented in Figure 5.1. The first step of this process is to identify the critical modes of the test systems. Not all modes in the system share the same level of importance. In certain situations one mode or another will become the critical mode of the system and the magnitude of the real part of that critical mode will clearly indicate the proximity of the system to voltage collapse. To find this critical mode in the test systems used in this thesis, first the load at a bus is incrementally increased until the system becomes unstable. At the point of instability the modal content of the system is analyzed and the mode whose eigenvalue has zero for its real component is identified as the critical mode of the system. In the test systems used in this thesis there was typically just one critical mode. However, if there were multiple critical modes the proposed analysis method would still work. There would simply be the possibility that multiple state variables would need to be Prony analyzed if the different critical modes participated most in different states.

After the critical modes have been identified the system is returned to its original loading and then the loading is incrementally increased again. This time at each step of the increase a time domain simulation is performed where a small disturbance is injected into the system to induce oscillations in the responses of the state variables. These responses are recorded for each state variable and are analyzed with Prony to extract their modal content. Also during each step the standard eigenanalysis is performed on the system to extract its actual modal content. This sequence of measurements produces a clear picture of the actual and estimated trajectories of the critical mode in the imaginary plane as it drifted towards the imaginary axis and system instability. The difference between the actual and estimated trajectories then provides some indication of the relative accuracy of Prony analysis on different state variables.

5.2 Experimental Setup

The following experiments were performed using a matlab script which implements the steps of the proposed analysis method. This script incorporates PSAT a matlab toolkit for power system analysis [7] and Pronytool a matlab toolkit for prony analysis [29]. The two test systems used in these experiments are the IEEE 9 and 14 bus test systems. These two systems were chosen because they represent simple but relatively different kinds of systems so that the results are diverse but fairly easy to interpret. The layout of the 9 bus system is pictured in Figure 5.2 while the layout of the 14 bus system can be found in Figure 5.3. PSAT's fourth order synchronous machine model is used to simulate the generators in both of these test systems. In addition, the excitation of these generators is modeled with the type 2 AVR model in PSAT which is the same as the standard IEEE model 1. The perturbations which induce the ring down needed by the Prony analysis are induced with a slight temporary adjustment to the reference voltage of the AVR on generator 1.

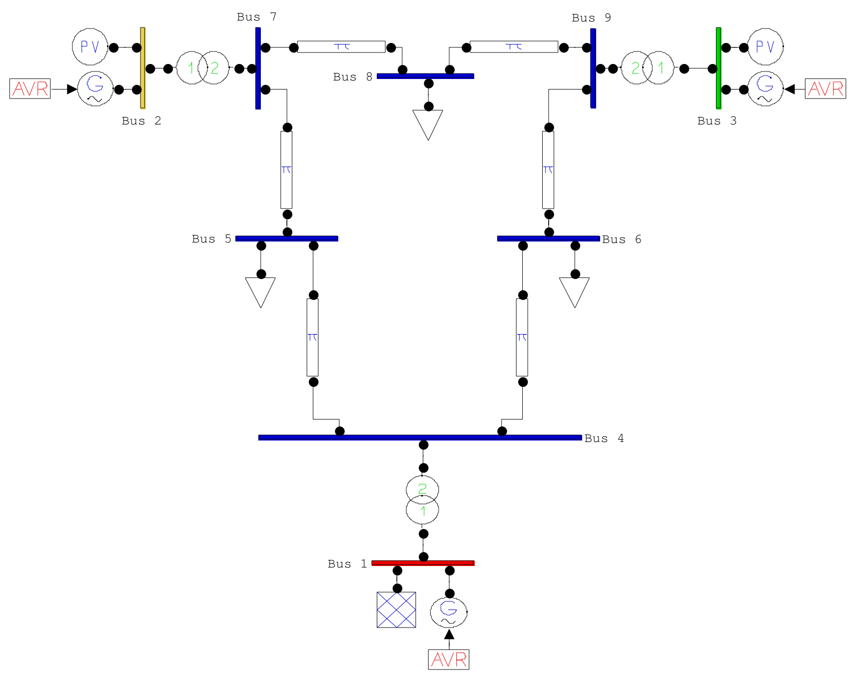


Figure 5.2: IEEE 9 Bus Test System.

5.3 9 Bus Results

5.3.1 Eigenanalysis

Table 5.1 contains the eigenanalysis results for all of the system modes from the beginning, middle and end of the incremental load increase. From these results it is clear that only one of the modes develops a clear trajectory towards the imaginary axis as the load is increased. The other modes either move away from the imaginary

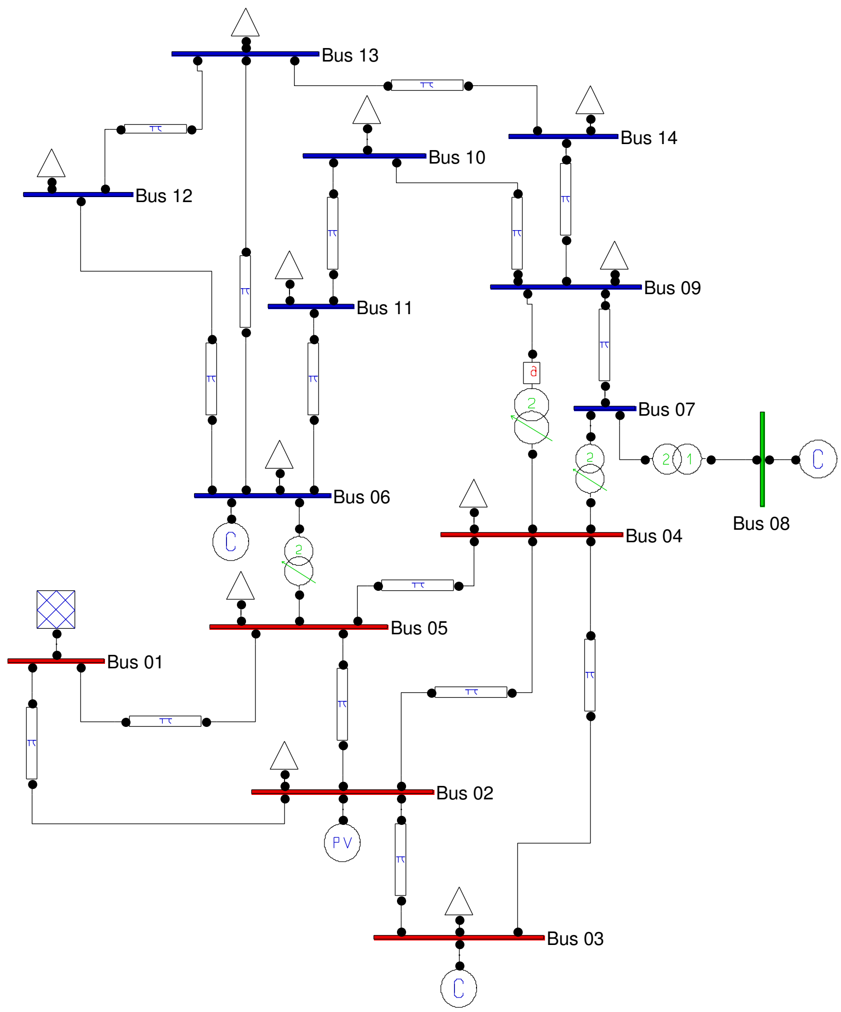


Figure 5.3: IEEE 14 Bus Test System.

axis or hover in place. Its also significant to note that though the real component of the eigenvalues can change significantly, their imaginary component and frequency (which is related to the imaginary component) do not change much. As a result, it is possible to identify the critical eigenvalue by its imaginary component or frequency.

Table 5.1: Eigenvalue progression due to increased loading at bus 6. (The critical mode is highlighted)

Eigenvalue Identifier	Loading	Real	Imaginary	Frequency
4	3.5 (min)	-0.93276	11.23694	1.788410364
	3.75	-0.96509	11.13464	1.772128852
	3.975 (max)	-1.00194	10.99104	1.749274255
6		-0.31505	7.25405	1.154515215
		-0.40849	7.01104	1.115839063
		-0.68353	6.54839	1.042206201
8		-3.81886	7.21153	1.147747963
		-3.77926	7.15435	1.138647504
		-4.26919	7.62884	1.214164757
10		-4.07028	7.48697	1.191585498
		-4.10212	7.48941	1.191973835
		-4.13622	7.49387	1.192683664
12		-4.19191	7.59306	1.208470206
		-4.22323	7.60921	1.211040553
		-3.67474	7.04181	1.120736249
15		-0.24167	1.91613	0.304960848
		-0.19333	2.16811	0.345064617
		-0.00519	2.65732	0.422924624
18		-0.26388	0.92569	0.147327795
		-0.27388	0.93078	0.148137892
		-0.28546	0.93495	0.148801566

5.3.2 Undirected Prony Analysis

Figures 5.4 and 5.5 show the actual trajectory as well as the estimated trajectories produced using each state variable in the system. In both graphs the actual trajectory is located roughly in the center of the largest clump of trajectories. However, these graphs are not meant to show the relative accuracy of each state variable. Instead, from these graphs we can see that the accuracy varies greatly from state variable to state variable. Therefore an arbitrary choice of response to analyze with Prony will not result in necessarily optimal analysis. In addition, from these graphs we can also see that as the system approaches instability the accuracy of all of the

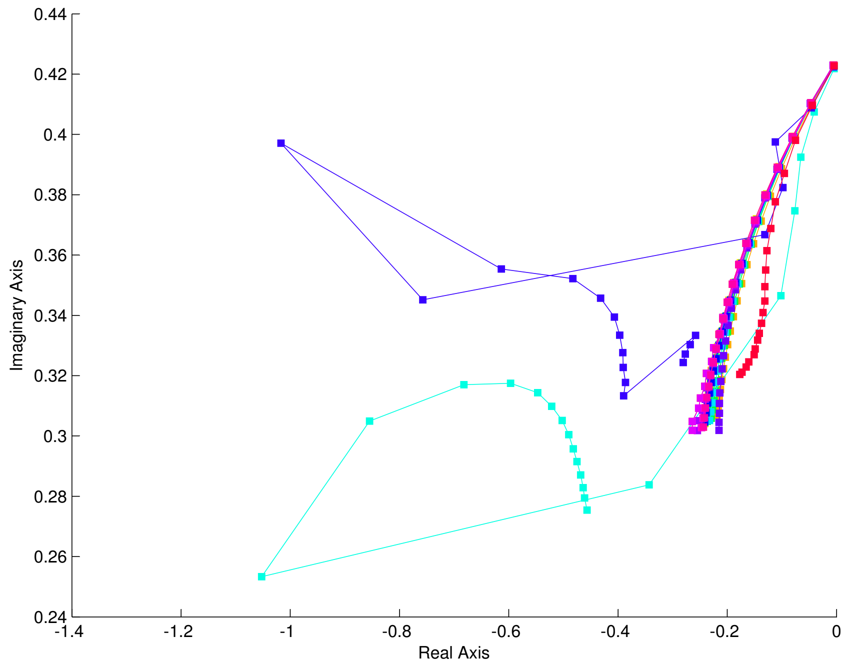


Figure 5.4: Results of Prony analysis, with 100 modal order, on all state variables versus eigenanalysis.

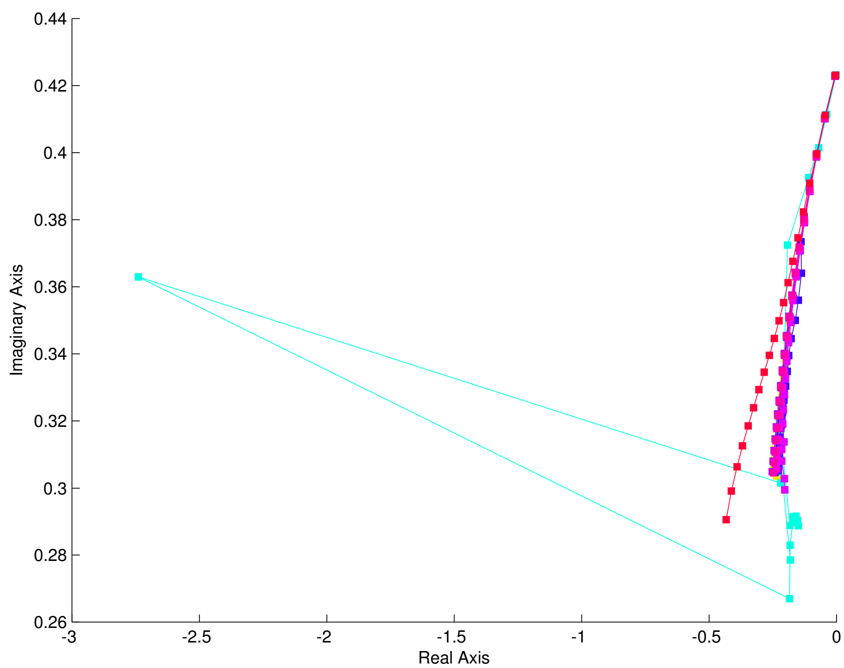


Figure 5.5: Results of Prony analysis, with 200 modal order, on all state variables versus eigenanalysis.

state variables converges. However, far away from the instability the different state variables have very different estimates for the value of the critical mode. Therefore, though all of the state variables will provide a last minute warning of voltage instability, some of them may not accurately represent the trajectory earlier on when operators would have had more time to act.

The modal order of the Prony analysis has a significant effect on the accuracy of the resulting trajectories. Increasing the modal order improves the accuracy of the Prony analysis of many of the state variables but it also greatly decreases the accuracy of several of those state variables. So, increasing the modal order for Prony does not always result in increased accuracy. This is similar to the effect of modal order on Prony accuracy noted in [24]. In addition, increasing the modal order greatly increases the complexity of the Prony calculation resulting in higher computing costs.

5.3.3 Participation Factor Analysis

Table 5.2: Participation Factor values and associated states for the critical eigenvalue and summed across all eigenvalues.

		Mode in State		State in Mode	
Eigenvalue	Loading	Participation	Associated State	Participation	Associated State
15	3.5 (min)	0.225258425	'e1q_Syn_2'	0.498831612	'delta_Syn_2'
	3.75	0.212597749	'e1q_Syn_2'	0.499719555	'delta_Syn_2'
	3.975 (max)	0.212597749	'e1q_Syn_2'	0.499719555	'delta_Syn_2'
Total		0.52287626	'vr2_Exc_3'	2.876349285	'delta_Syn_2'
		0.523874284	'e1q_Syn_2'	2.673174512	'delta_Syn_2'
		0.523874284	'e1q_Syn_2'	2.673174512	'delta_Syn_2'

The participation factor analysis results presented in Table 5.2 show both the magnitude and associated state variable of the largest mode in state and state in mode participation factors. From this analysis it is clear that for both types of participation the state which is most associated with the critical mode remains constant throughout the increase in load. This means that the same signal should be used to obtain the best Prony analysis for the critical mode. This is an important result because if the optimal signal changed frequently then it would difficult to implement this method in the real world. As it is, though, a system could be set up to monitor just the one recommended state variable to warn if its associated critical mode were developing a trajectory. The same cannot be said of the total value though. So, just one state variable may not be suitable for the estimation of all of the critical modes in the system.

It is also interesting that the state variable suggested by the mode in state participation factor is the q-axis voltage of the second generator. This is a value associated with the field rotor circuits of that generator and would not be an obvious choice for the best response signal to analyze. The state in mode participation factor, on the other hand, suggests the change in the rotors relative angle as the signal that is best for Prony analysis. This is a much more intuitive choice.

5.3.4 Directed Prony Analysis

The estimated trajectories of only the state variables suggested by the participation factors versus the actual trajectory are shown in Figures 5.6 and 5.7. It is clear from these graphs that the estimated trajectory of the state variable suggested by the mode in state participation factor is more accurate than the trajectory of the

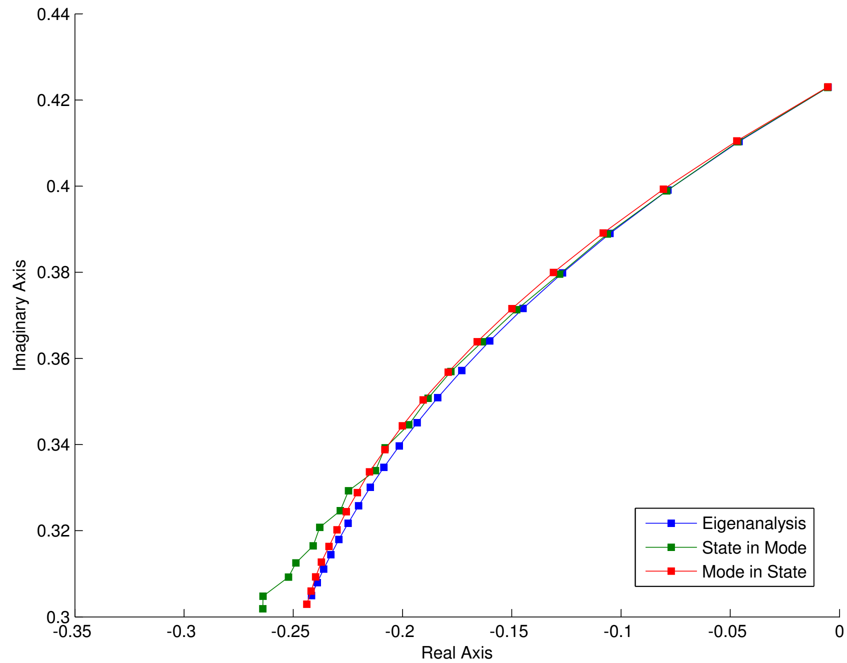


Figure 5.6: Results of Prony analysis, with 100 modal order, on participation factor suggested state variables versus eigenanalysis.

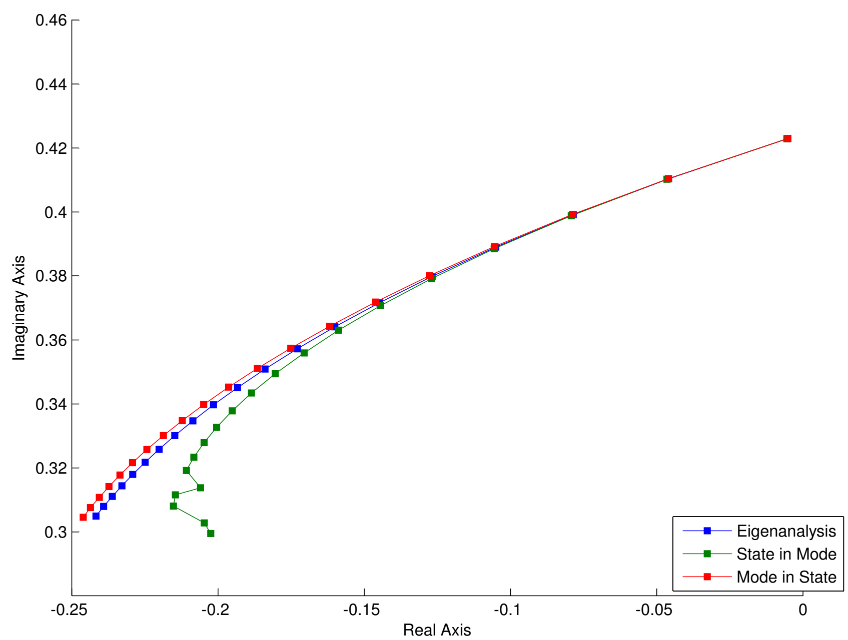


Figure 5.7: Results of Prony analysis, with 200 modal order, on participation factor suggested state variables versus eigenanalysis.

state variable suggested by the state in mode participation factor. In particular, the mode in state trajectory is accurate much farther away from instability and shows a clear trend from the beginning. The state in mode trajectory in the 200 modal order case does not develop a clear trajectory until 6 data points into the load increase.

Another interesting feature of these graphs is that the state in mode trajectory reacts extremely to the change in mode while the mode in state trajectory hardly changes at all. This indicates that the state variable suggested by the mode in state participation factor can be accurately analyzed using a lower modal order than other signals in the system. So, utilizing the mode in state participation factor to identify the signal to analyze could possibly reduce the amount of computer power needed to perform the analysis. Because the state in mode trajectory appears to hop across the actual trajectory it seems as though selecting a modal order between 100 and 200 will result in an more accurate Prony estimate. However, this is not the case.

5.3.5 Intuitive Prony Analysis

It is tempting to suggest that simply selecting the intuitive state variables such as the rotor velocity or the rotor angle will produce acceptably accurate Prony analysis estimates. This seems to be supported by the state in mode participation factor which suggests the rotor angle of generator 2. The estimate which results from that suggestion is not the most accurate but it is still fairly accurate. However, if we were to simply choose some intuitive state variable at random to perfrom the Prony analysis on, the results are potentially much less accurate. Figure 5.8 shows the

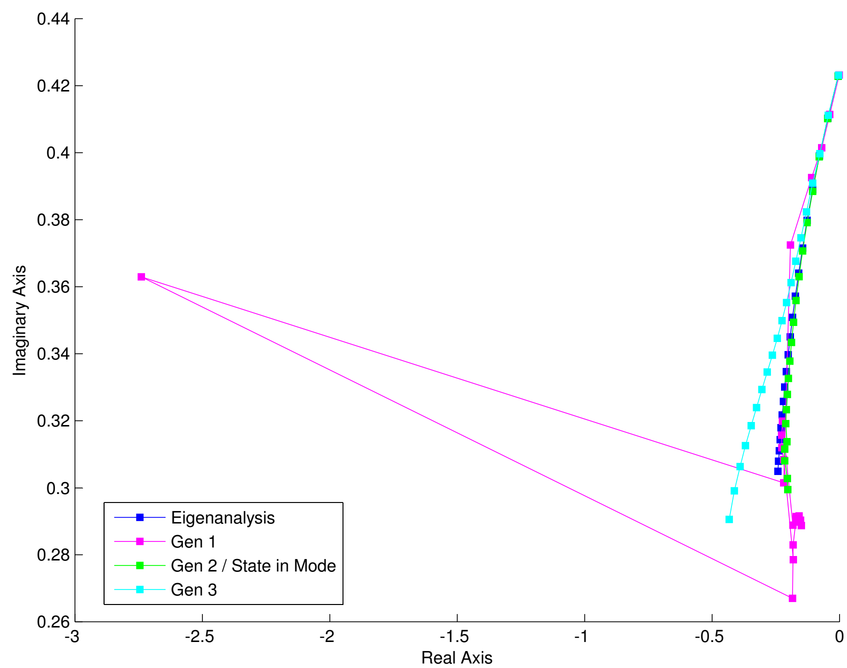


Figure 5.8: Results of Prony analysis, with 200 modal order, on the rotor angle state variables of all of the generators versus eigenanalysis.

estimates that are generated by performing a Prony analysis on the responses of the rotor angles for all of the generators in the system. For this graph it is clear that the rotor angle does not always produce an accurate estimate. In fact, the rotor angle response for generator 1 produces one of the least accurate estimates of any state variable in the entire system. Even averaging all of the estimates from the different rotor angle responses does not produce an estimate as accurate as the one that is produced using the state variable suggested by the mode in state particiaption factor.

Table 5.3: Eigenvalue progression due to increased loading at bus 9. (The critical mode is highlighted)

Eigenvalue Identifier	Loading	Real	Imaginary	Frequency
12	0.213 (min)	-0.9699	8.34296	1.327820219
	0.473	-1.08103	8.08165	1.286231538
	0.733 (max)	-1.40943	7.71607	1.22804781
14		-1.75005	6.69539	1.065601923
		-1.50339	7.01444	1.116380188
		-1.08142	7.46784	1.188540871
16		-0.39176	7.11032	1.131639929
		-0.36334	7.09952	1.129921059
		-0.35278	7.08416	1.127476445
18		-0.50438	6.5877	1.048462567
		-0.60012	6.49758	1.034119557
		-0.72852	6.30335	1.003206965
22		-0.56571	2.06121	0.328050993
		-0.39472	2.0142	0.320569137
		-0.02643	2.08307	0.331530112
24		-1.02663	1.62767	0.25905112
		-0.4011	1.62718	0.258973135
		-0.56847	1.68709	0.268508085
26		-0.35816	1.4741	0.234609753
		-1.02302	1.62856	0.259192768
		-1.01869	1.62907	0.259273937
28		-0.52922	0.69129	0.110021963
		-0.53173	0.70092	0.111554622
		-0.53617	0.70977	0.11296314
32		-0.56049	0.41282	0.06570219
		-0.56842	0.40884	0.065068755
		-0.57737	0.40225	0.064019926
39		-1.0039	0.00012	1.91E-05
		-1.0039	0.00016	2.55E-05
		-1.00389	0.0002	3.18E-05

5.4 14 Bus Results

5.4.1 Eigenanalysis

The results of the eigenanalysis at the beginning, middle and end of the incremental load increase are similar to the results from the 9 Bus system. These results are presented in Table 5.3. Again, we see a single critical mode that develops a clear trajectory while the rest of the system modes do not move or move away from, not toward, the imaginary axis. Also, as was the case with the 9 bus system, the frequencies of the modes do not change much so they can be used to identify the different modes in the system. One difference between the behavior of the 14 bus modes and the 9 bus modes is that there was much more modal movement in the 9 bus system. Most of the modes in the 14 bus system don't really change their values much at all over the course of the load increase.

5.4.2 Undirected Prony Analysis

The trajectories of the critical mode produced by all of the system state variables can be seen in Figures 5.9 and 5.10. Similar to the 9 bus system, when the estimated trajectories are graphed, we can see that the trajectories all converge just before the point of instability. However, increasing the modal order of the Prony analysis has a much more dramatic affect on the accuracy of the trajectories for the 14 bus system than it did for the 9 bus system. Many of the 20 modal order trajectories are obviously significantly inaccurate for most of the load increase.

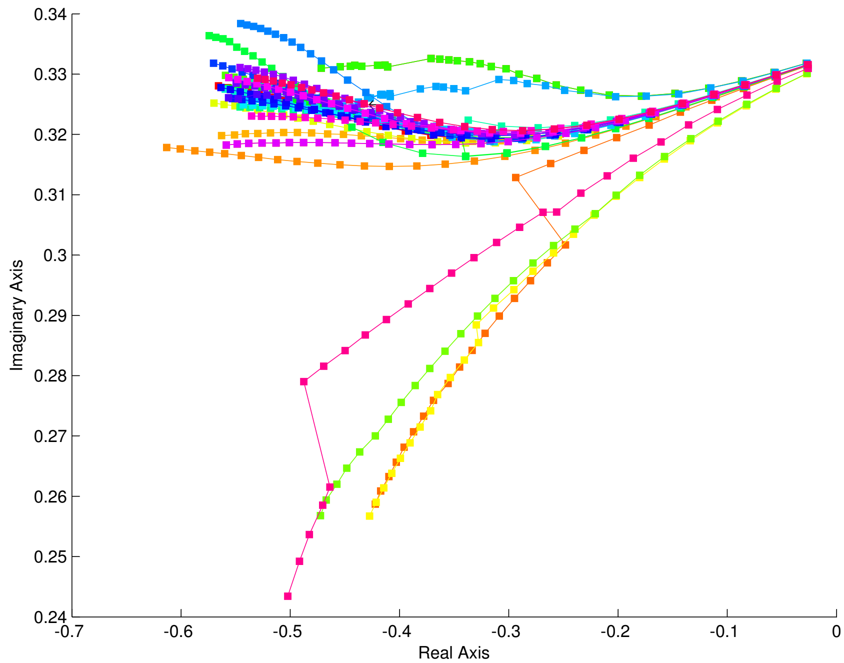


Figure 5.9: Results of Prony analysis, with 20 modal order, on all state variables versus eigenanalysis.

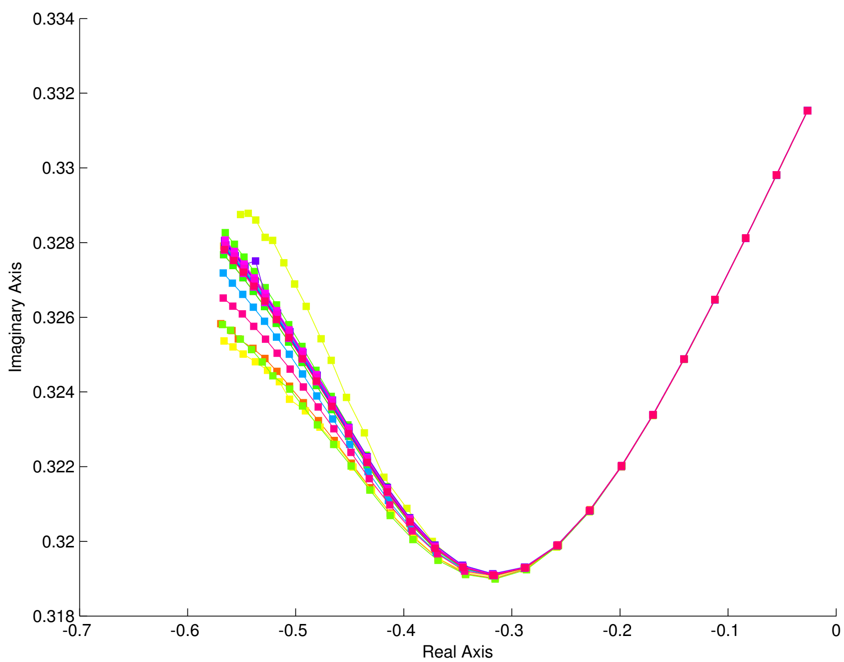


Figure 5.10: Results of Prony analysis, with 40 modal order, on all state variables versus eigenanalysis.

Meanwhile, the 40 modal order trajectories all converge long before the point of instability. Still, from both of these sets of trajectories it can be seen that the accuracy of the estimated trajectory varies greatly between different state variables. Therefore, selection of the correct state variable is essential to produce the most accurate estimate.

Significantly lower modal orders were used for the Prony analysis in these experiments because the higher modal orders produced highly inaccurate estimates. This effect where high modal orders yield less accurate results was also observed in [24].

5.4.3 Participation Factor Analysis

Table 5.4: Participation Factor values and associated states for the critical eigenvalue and summed across all eigenvalues.

Eigenvalue	Loading	Mode in State Participation	Associated State	State in Mode Participation	Associated State
22	0.213 (min)	0.125950728	'e1q_Syn_1'	0.191486474	'delta_Syn_1'
	0.473	0.161258685	'vf_Exc_4'	0.161309009	'delta_Syn_1'
	0.733 (max)	0.125950728	'e1q_Syn_1'	1.120077673	'delta_Syn_1'
Total		0.569581845	'e1q_Syn_1'	1.120077673	'delta_Syn_1'
		0.553244828	'vf_Exc_4'	1.072298372	'delta_Syn_1'
		0.569581845	'e1q_Syn_1'	0.191486474	'delta_Syn_1'

The participation factor analysis for the 14 bus system presented in Table 5.4 is less coherent than was the case for the 9 bus system. The most significant difference is that the state variable suggested by the mode in state participation factor changes in the middle of the load increase before going back to its original value. However a more in depth analysis revealed that the participation factor only suggested these two state variables throughout the load increase. As we will see in the next section both signals provide accurate estimates for the entire trajectory of the critical eigenvalue. So it is reasonable to select either of the state variables. Still the shift in suggested state variables is not ideal. One possible explanation for this behavior could be that there many state variables in this system which are all roughly equal in terms of their suitability.

Regardless of which one you pick, however, the suggested state variables of the mode in state participation factor are again not intuitive choices. Meanwhile, the state in mode participation factor once again suggests the logical choice of rotor angle.

5.4.4 Directed Prony Analysis

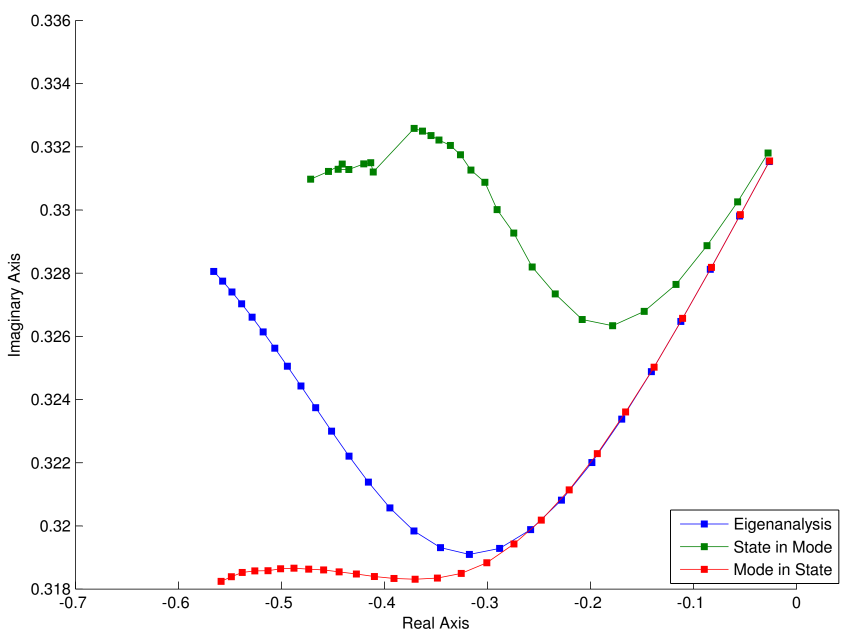


Figure 5.11: Results of Prony analysis, with 20 modal order, on participation factor suggested state variables versus eigenanalysis.

Figures 5.11 and 5.12 show only the estimated trajectories generated using the

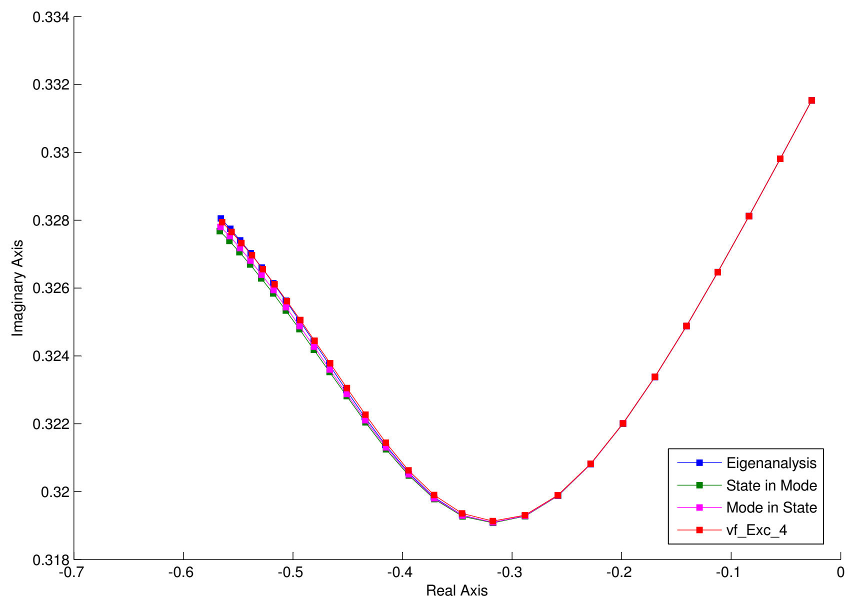


Figure 5.12: Results of Prony analysis, with 40 modal order, on participation factor suggested state variables and vf_Exc_4 versus eigenanalysis.

Prony analysis of the participation factor suggested state variables versus the actual trajectory of the critical mode. Again, the change in modal order affected these results much more than it affected the results for the 9 bus system. In both cases though, the mode in state suggested state variable's trajectory is more accurate than the state in mode trajectory. In the 20 modal order case the difference in accuracy is significant indicating again that the mode in state suggested state variable can produce accurate estimates at lower modal orders than other state variables. The difference in the 40 modal order case is much smaller but is still evident. Finally, the trajectory generated by the vf_Exc_4 state variable can also be seen to be very accurate as well. At the beginning of the loading process its estimate of the mode seems more accurate. However, it is more accurate in terms of the imaginary component but less accurate in terms of the real component until about a quarter of the way through the loading. This seems to fit with the observed shift in mode in state suggested state variables.

5.5 Discussion

These results clearly show that mode in state participation factors provide useful direction regarding the selection of which state variable response to Prony analyze in order to maximize accuracy. This provides some confirmation of the hypothesis that mode in state participation factors are better suited to monitoring applications due to the nature of the information that they contain. Choosing to analyze the intuitive state variables such at the rotor angles of the generators have been shown to provide less accurate results. In fact, it is possible that choosing the an intuitive state variable could result in a very inaccurate and confusing result. This may explain some of the difficulty that has been observed in the literature regarding the effective use of Prony analysis [24]. Using mode in state participation factors to select the signal to analyze simplifies the process of applying Prony to power system responses and improves the results.

It is important to note that the state variables which make up the mathematical models used in these experiments do not always correspond directly to components in real world machines [16]. Therefore, the field voltage that was suggested in the 9 bus test system experiment may not directly refer to a measurable voltage in the generator which that synchronous machine model represents. However, that state variable does correspond to some feature of that generator and so provides some good guidance on which real world signals should be analyzed with

Prony.

With mode in state participation factor suggestions on which state variables to analyze, Prony analysis could be used to accurately estimate the modes of a real world power system. This analysis could be used to provide a reliable on-line relative assessment of the system's proximity to instability. This application of participation factors may help to address the accuracy issues that have limited the use of Prony analysis in the past. To the best of our knowledge, this is the first time participation factors have been suggested as a means to improve the Prony analysis of power systems for stability assessment.

Chapter 6: PSS Placement

The field of power system stabilizer (PSS) tuning and placement is large area of research in the power engineering community. This thesis is not meant to address the tuning aspects of PSSs and does not directly propose a new method for PSS placement. Instead this thesis reexamines an existing technique for PSS placement in the light of the recent development of new definitions for participation factors. Through this reexamination, this thesis hopes to provide numerical evidence for the potential applications of the two types of participation factors and to explain some past difficulties regarding the use of participation factors for PSS placement. This chapter is included to provide some brief background on the structure and operation of PSSs as well as the problem of their placement to provide the reader with the necessary information to understand the experiments presented later in this thesis.

Power system stabilizers were introduced to address the rotor-angle instability problem that resulted from the widespread adoption of static excitation systems and long-distance transmission lines in multi-machine power systems [14, 30]. The rudimentary function of a PSS is to extend the rotor-angle stability limits of a power system. It accomplishes this by providing supplemental damping to the rotor oscillations of a generator by adjusting the generator's excitation.

When a disturbance occurs in the power system an imbalance is created between the mechanical torque acting on the rotor of a generator and the electro-magnetic torque opposing that motion. As a result, the angular velocity and electrical power output of the generators will fluctuate around their steady-state values. The swing equation below is used to describe the effect of such an imbalance.

$$\frac{2H}{\omega_0}\frac{d^2\delta}{dt^2} = \overline{T_m} - \overline{T_e} - K_D \triangle \overline{\omega_r} \tag{6.1}$$

In this equation δ is the rotor angle, ω_r is the angular velocity of the rotor, ω_0 is the rated value for the angular velocity of the rotor, H is the inertia constant of the generator, T_m is the mechanical torque, $T_e$is the electro-magnetic torque, and $K_D$is the damping coefficient. This equation tells us that when a generator is disturbed, its rotor will accelerate at a rate that is proportional to the net torque divided by the inertia constant. [16, 30]

For small disturbances the change in the value of the electro-magnetic torque (T_e) is described by the following equation

$$\Delta T_e = T_S \Delta\delta + T_D \Delta\omega \tag{6.2}$$

where T_S is the synchronizing coefficient for the generator and $T_D$is the damping coefficient. From this equation it can be seen that the change in the electro-magnetic torque is the result of the change in rotor angle multiplied by some synchronizing coefficient plus the change in rotor velocity multiplied by some damping coefficient. The synchronizing torque portion of the equation captures the tendency of the generator to oppose deviations in its angle from equilibrium and to stay in synchrony with the rest of the system. From this equation and equation 1 it can be understood that a positive synchronizing component will create a net decelerating torque in the event of a change in rotor angle. This will cause the generator to slow down until the rotor angle has been restored to its equilibrium value. Similarly, the damping torque part of the equation represents the generators tendency to oppose changes in its velocity away from some stable equilibrium point. As long as both the synchronizing and damping coefficients are sufficiently positive and large, the system will return to its steady-state operating point following a disturbance. [16,30]

The value for $T_D$is influenced by many factors including the design of the generator, the settings of its excitation system and the strength of its connection to the power network. During disturbances the value for $T_D$can be reduced significantly, resulting in unacceptably small damping or in some extreme cases amplification of the oscillations. In these extreme cases, the amplification will cause the oscillations to grow eventually resulting in a loss of synchronism.

Adding a power system stabilizer increases the generator's damping coefficient and allows it to continue operating under conditions where it would have not had enough natural damping. The power system stabilizer counters oscillations by changing the input to the generator's excitation at just the right time to oppose the oscillations. The reduced damping that occurs during disturbances is the result of phase lags that are caused by the field time constants and the lags in the normal voltage regulation loop. In other words these controls take time to act and that can put their actions slightly out of sync with the signal that caused it. The PSS uses phase compensation to adjust the timing of the correction signal so that it correctly opposes the oscillations that are detected in the generator's rotor. [16, 30]

6.1 Structure and Design of PSSs

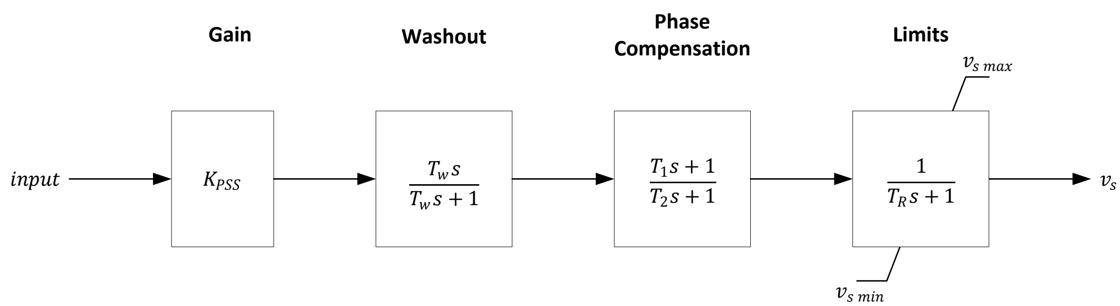


Figure 6.1: Example Structure of a Power System Stabilizer.

The PSS represented in Figure 1 is comprised of four blocks: a gain block, a washout block, a phase compensation block and a limiter block. The gain block determines how much damping will be introduced by the PSS. The washout block is essentially a high-pass filter that allows steady changes in the angular velocity to not affect the PSS. This is so that the PSS will only react to actual disturbances and not to normal adjustments. The phase compensation block provides a phase-lead characteristic to compensate for the phase lag that occurs between the exciter input and the generator torque. Finally, the limiter block determines the maximum and minimum output signals that can be produced by the PSS.

The gain and time constant values of this PSS (K_{PSS}, T_w, T_1, T_2) must be chosen with care by a process referred to as tuning. Tuning is difficult because the PSS must meet conflicting requirements by providing damping for both local and inter-area oscillations under both small-signal and transient conditions. Failure to properly tune a PSS can result in it amplifying oscillations rather than damping them, contributing to the problem rather than solving it. As a result of this difficulty many methods have been proposed for producing the optimal values for the power system stabilizer [14]. However, this work is focused on the optimal placement of PSSs and does not incorporate any advanced tuning methods.

6.2 Input Signals

In addition to selecting the proper gain and time constant values for the PSS, the type of input that the PSS will use also needs to be specified. PSSs have been proposed that utilize several different inputs including changes in rotor velocity, generator power, frequency, and generator voltage. The input that should be used to achieve ideal PSS performance is a matter of debate in the literature. Each type of input has its advantages and disadvantages.

The most logical choice for input is the rotor velocity because the primary purpose of the PSS is to oppose rotor oscillations. As a result, rotor velocity has been frequently promoted as a PSS input in the literature. However, this input can excite torsional modes in some situations introducing a new source of instability [16].

The electrical power signal of the generator is closely related to its rotor velocity so it also makes a logical choice for PSS input. Furthermore, the electrical power signal of the generator has the advantage that it does not contain the torsional modes that can cause problems for the rotor velocity input. The disadvantage of the power-input PSS is that it can produce spurious outputs during load changes because the mechanical power of the generator is not being considered [16].

Frequency-input PSSs can achieve better damping of inter-area oscillations because the frequency signal is more sensitive to those oscillations. The downside of frequency input is that the frequency signal often contains noise from the power system which can make it unusable. In addition, the frequency input also has the same torsional mode problem as the rotor velocity input [16].

Finally, generator voltage is not a class of input that is discussed much in the literature but it has been included as an option in the PSAT toolkit used by this thesis presumably for research purposes [7].

6.3 Optimal PSS Placement

It is neither economical nor necessary to equip every generator in a power system with a PSS [31]. Therefore, a decision needs to be made about which generators in the system should be equipped with a PSS in order to achieve the maximum stability enhancement. The significance of this problem can be seen in the large number of methods that have been proposed for determining the optimal placement of PSSs. One method used the sensitivities of the different generators mechanical-mode damping to the gain of the PSS to pick the most effective generator for stability enhancement [32]. Another selects the locations based on the sensitivities of the real parts of the eigenvalues of the system to a gain that is determined by a ratio of the generator's flux and speed [33]. Coherent groups, a method that has been useful in transient stability studies, have also been used to determine optimal placement locations [34]. Finally, participation factors are another method that has been suggested as a way to identify the optimal sites for PSS placement [14].

This thesis is particularly concerned with using participation factors to select the optimal PSS locations. In the literature the participation factors that have been used to select placement locations have been computed in two ways. Some of the prior work utilized participation factors that were calculated from the reduced power flow Jacobian of the system only [6], while other work has utilized participation factors that were calculated from the system state matrix [1, 14]. However, in both cases, the original participation factor definition was used which may have led to some incorrect results. This can be seen in the results presented in [31] where the participation factors clearly do not indicate the optimal location for the PSS. This thesis explores the possibility that some of these confusing results were due to the participation factor definition that was being used. The new state in mode participation factor definition presents an opportunity to revisit this placement method and account for some of the previously confusing results.

Chapter 7: State in Mode Participation Factors

Power system stabilizers are an important tool of power system engineers for extending the rotor-angle stability limits of systems. However, they can be a costly investment for power companies and so placing them on every generator in the system is not an economical option. A decision must be made about which generators should receive PSSs in order to maximize the stability enhancement for the entire system.

Some previous work has shown that participation factors can provide a useful indicator of the critical locations of the system [14]. But confusing results have also been observed regarding the use of participation factors to determine optimal PSS locations [1, 31]. The recent discovery of the dichotomy of participation factors provides a possible explanation for these confusing results.

State in mode participation factors are intuitively well suited for control applications because they provide information about which states most affect the modes of the system. Increasing the stability of a system has a lot to do with improving the damping of certain troublesome modes of that system. Therefore, to achieve the optimal increase in stability it makes sense to implement controls for those states which have the greatest effect on a troublesome mode. It should be said that there is currently no clear direct relationship between participation factors and control siting from a theoretical perspective. However, the state in mode participation factors are identifying states that are particularly sensitive to perturbations and this may indirectly indicate locations that produce the most significant results from the addition of controls.

7.1 Proposed PSS Placement Technique

To evaluate the effectiveness of state in mode participation factors for control applications we revisit the power system stabilizer placement problem that has been addressed frequently in the literature [1, 14, 31–33, 35]. Our approach closely resembles the approach used in [1,14]. The key difference between this work and the previous work is that instead of using the original definition for the participation factors, we use the new definitions and compare the placements suggested by the two types. For our experiments, the placements suggested by the mode in state participation factors correspond to the placements that would have been suggested in the original PSS placement work using participation factors. In addition, like [1] we use the entire system state matrix to generate our participation factors instead of just the reduced Jacobian that was used in [14].

The experiments begin with an analysis of the modes of each test system to determine which modes develop clear trajectories towards the imaginary axis as a result of an increase in load. As was pointed out in Chapter 5, not all modes in the system develop such a trajectory and so some are more important than others. In

Chapter 5 we focused on those modes as they indicated the proximity of the system to voltage collapse. In this chapter we will focus on the critical modes that develop trajectories because they are the troublesome modes that need to be corrected in order to preserve the stability of the system. We identify the critical modes for the test systems in this chapter in the same way we did for Chapter 5. We gradually increase the loading of the system at a bus and check the real and imaginary values of each mode at each step. The magnitude of the real component of a critical mode will noticeably decrease as the load is increased.

Once the critical modes are identified, we perform participation factor analysis to determine the participations of the generator related states in the modes of the system. These participations are then used to determine the predicted optimal placement for PSSs in the system. The generator that has the state variables with the highest participation factor for the critical modes is suggested as the optimal location for PSS placement.

To test these placements time domain simulations are then performed where a short fault is applied to a bus in the system. The damping of the response of the system to this fault is then examined for each possible placement of PSSs including the case where no PSS is used. The optimal location for a PSS will produce the most rapid damping of the system response to the fault. So, by comparing the time it takes for the systems with the different PSS placements to return to some steady-state, it is possible to determine which placement is best.

In addition, it is also possible to observe the improved damping of the system by examining the eigenvalues of the systems with different PSS placements. The systems with improved damping will have a noticeable increase in the magnitude of the real component of at least one of their eigenvalues. This analysis is also performed to further evaluate the optimal placement.

7.2 Experimental Setup

As was the case with the mode in state participation factor experiments, PSAT was used to perform these experiments. The IEEE 9 bus test system was also used again for this set of experiments. However, instead of the IEEE 14 bus test system, the 2 area test system featured in [16] was used as an additional example. This system is pictured in Figure 7.1. The 2 area system was used because it was used in [1] for PSS placement. In addition, this test system is a classic example that has been used in numerous books and papers and is well understood. This made it an excellent initial test system to evaluate to appropriateness of the new state in mode participation factors for PSS placement. The original time domain simulations from [1] as used in this investigation to prevent any discrepancy in parameters since the original test files were not available and the time domain simulations were performed with a simulator other than PSAT. In addition, the PSS tuning parameters presented in [1] are used for the new eigenanalysis and participation factor analysis work presented in this chapter.

The IEEE 9 bus system that is used for these experiments differs slightly from the version used in Chapter 5. In order to observe oscillations that did not rapidly damp even when a PSS was not used, it was necessary to push the system into an unstable state. As a result, the loading on the version of the 9 bus system used for these experiments is much higher than the loading on the 9 bus system used in Chapter 5. This difference in loading accounts for the slightly different participation factor analysis as the system is in a significantly different region of operation for these experiments. To establish the critical mode of the modified 9 bus system the same sequential load increase process was used that was used in Chapter 5.

Additionally, the PSSs that have been added to the 9 bus system needed to be tuned for that system. This was accomplished with a basic trial and error search of the PSS parameters to find settings that produced a reasonably good level of damping. Tuning was performed independently for each placement location and for each possible input. The parameters used for the velocity input PSSs which are primarily utilized in this work are shown in Figure 7.3. The same parameters were found to be most effective for all of the placement locations. These parameters do not necessarily represent the optimal tuning of the PSS for this system but they provide sufficient damping for the purposes of this study.

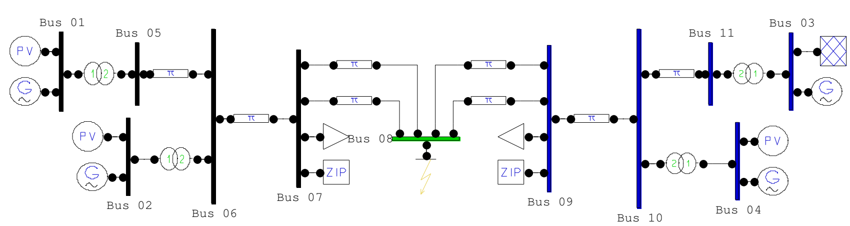


Figure 7.1: 2 Area Test System (area 1 is black, area 2 is blue).

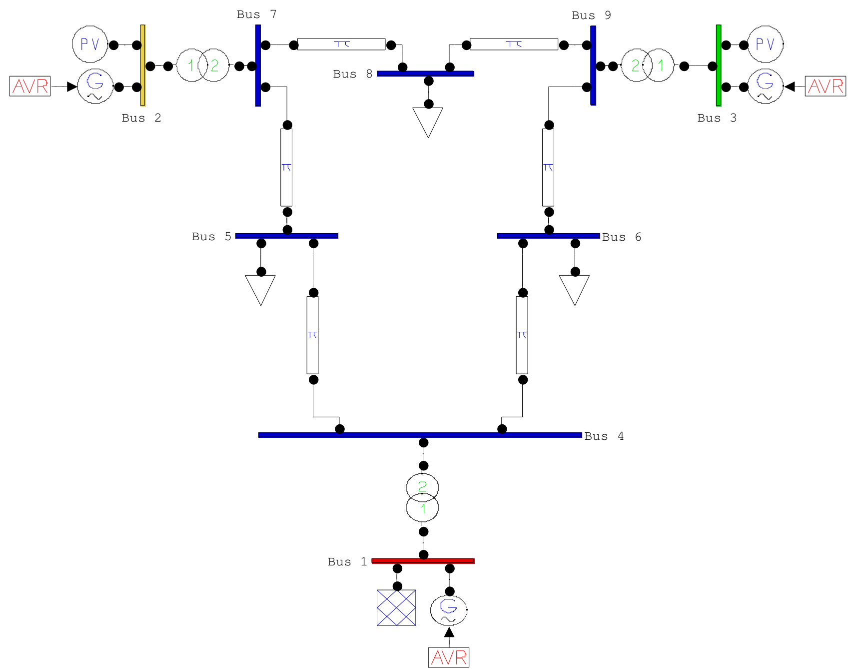


Figure 7.2: IEEE 9 Bus Test System (included again for reference).

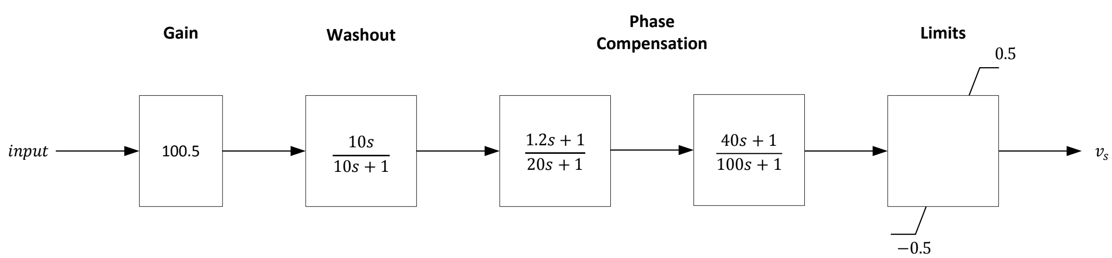


Figure 7.3: PSS Diagram including values used in the following experiments.

7.3 2 Area Test System Results

In his 2011 paper [1], Devedra Parmar presents a study of the optimal power system stabilizer placement for the 2 area system presented in Kundur's Power System Stability book [16]. In this study he presents the participation factor analysis of the system as well as the time domain simulations of the response of the 2 area system to a fault with PSSs placed in different locations. The time domain experiments in this section are drawn directly from Parmar's original work in an effort to preserve the placement results he reported.

7.3.1 Eigenanalysis

Table 7.1 presents the eigenanalysis results for the system modes of the 2 area test system at the beginning, middle and end of the incremental load increase. From these values we can see that two of the three modes develop trajectories towards the imaginary axis as their real values have a relatively large positive drift. Of these the 1.14 Hz mode (Eigenvalue 5) moves significantly more than the 0.5 Hz mode (Eigenvalue 11). This seems to indicate that the 1.14 Hz mode is the most important mode of this system while the 0.5 Hz mode is less important but still plays a roll in the stability of the system. This is interesting because the 0.5 Hz mode is understood to be an inter-area mode of this system while the two higher frequency modes are local-area modes. So, while both categories of mode are important, it is a local-area mode that is the most important. Furthermore, it is also interesting that only one of the two local-area modes develops a trajectory. This seems to indicate that placing a PSS in one of the two areas will have a significant greater effect than the other.

It is also important to note that the frequency values for the modes also drift in this analysis. However, the ordering of the modes remains constant so it is possible to identify them despite the minor changes in frequency.

Table 7.1: Eigenvalue progression due to increased loading at bus 9. (The critical modes with clear trajectories are highlighted)

Eigenvalue Identifier	Loading	Real	Imaginary	Frequency
5	13.67	-0.96336	7.1407	1.1468
	15.67	-0.8118	7.3475	1.1765
	17.67	-0.72408	7.3645	1.1777
	Drift	0.23928	0.2238	0.0309
7		-0.72224	7.256	1.1605
		-0.74339	7.4079	1.1849
		-0.73798	7.4367	1.1894
	Drift	-0.01574	0.1807	0.0289
11		-0.33902	3.0665	0.49103
		-0.27361	3.2494	0.51898
		-0.24435	3.2636	0.52087
	Drift	0.09467	0.1971	0.02984

7.3.2 Participation Factor Analysis

Table 7.2 presents the original participation factor analysis from Parmar's paper along with the participation factor analysis performed using the new definitions. Also, the participation factor for the rotor angle is included in addition to the rotor velocity participation factors which were used by Parmar. From this table it is clear that our test system is very similar to the one used by Parmar because the mode in state participation factors for the rotor velocity for our system match closely the values reported by Parmar.

Table 7.2: Participation Factor values for the rotor velocity from [1] compared with the values of the mode in state and state in mode participation factors for the rotor velocity and angle from our test system. (The largest participation factors are highlighted for each set of generators)

Parmar 2 Area Values						
	Mode in State					
Mode Frequency	Gen 1	Gen 2	Gen 3	Gen 4	Gens 1+2	Gens 3+4
0.54 Hz	0.1442	0.0511	0	0.1903	0.1953	0.1903
1.05 Hz	0.0032	0.2494	0	0.0085	0.2526	0.0085

Rotor Velocity Participation Factor Magnitude						
	Mode in State					
Mode Frequency	Gen 1	Gen 2	Gen 3	Gen 4	Gens 1+2	Gens 3+4
0.5 Hz	0.1456	0.0496	0.0898	0.191	0.1952	0.2808
1.15 Hz	0.0031	0.2499	0.196	0.0084	0.253	0.2044
1.16 Hz	0.256	0.0066	0.0039	0.191	0.2626	0.1949
	State in Mode					
Mode Frequency	Gen 1	Gen 2	Gen 3	Gen 4	Gens 1+2	Gens 3+4
0.5 Hz	0	0	0	0	0	0
1.15 Hz	0	0	0	0	0	0
1.16 Hz	0	0	0	0	0	0
Total	0	0	0	0	0	0

Rotor Angle Participation Factor Magnitude						
	Mode in State					
Mode Frequency	Gen 1	Gen 2	Gen 3	Gen 4	Gens 1+2	Gens 3+4
0.5 Hz	0.1456	0.0496	0.0898	0.191	0.1952	0.2808
1.15 Hz	0.0031	0.2499	0.196	0.0084	0.253	0.2044
1.16 Hz	0.256	0.0066	0.0039	0.191	0.2626	0.1949
	State in Mode					
Mode Frequency	Gen 1	Gen 2	Gen 3	Gen 4	Gens 1+2	Gens 3+4
0.5 Hz	0.2256	0.2143	0.2857	0.2744	0.4399	0.5601
1.15 Hz	0.0365	0.4636	0.4279	0.0719	0.5001	0.4998
1.16 Hz	0.434	0.1	0.066	0.4	0.534	0.466

The participation factors from Parmar's paper suggest that placing PSSs on generators 1 and 2 in area 1 is the optimal placement. This is largely due to the 0 values for generator 3 in those results. As a result of these 0 values, both the 0.5 and 1.05 Hz modes discuss in Parmar's paper agree that area 1 is best location. However, the mode in state participation factors calculated for this work do not present such a clear conclusion.

In the mode in state participation factors calculated for this work, the values for generator 3 are not 0. As a result, the area with the largest participation factor for the 0.5 Hz mode was area 2 instead of area 1. Meanwhile, both of the local-area modes (1.15 and 1.16 Hz) indicate that area 1 is the better location. Since the 0.5 Hz and 1.15 Hz modes appear to be the most important due to their clear drift, we hypothesize that their participation factors should provide better information about where to place the PSSs. However, in the case of the mode in state participation factors these two modes strongly disagree on the optimal location.

The state in mode participation factors are 0 for the rotor velocity so the rotor angle was also considered. The mode in state participation factors for the rotor angle are identical to the mode in state participation factors for the rotor velocity. As a result, they still do not provide a clear suggestion about where to place the PSS in the system. However, the rotor angle state in mode participation factors present a less conflicted message. The 0.5 Hz modes participation factor still suggests placing the PSSs in Area 2 as was the case with the mode in state participation factors. But now the 1.15 Hz modes participation factors are evenly split between suggesting Area 1 or Area 2. Therefore, when considering the participation factors of both of the critical modes together, the suggestion is clearly to place the PSS in Area 2. This is the correct placement as we will see in the next sets of results. If we consider only the 1.15 Hz mode, since it is the most critical, we get a confusing picture because the participation factors for the placements are so equal. So, considering both of the critical eigenvalues seems to be necessary.

7.3.3 PSS Placement Time Domain Results

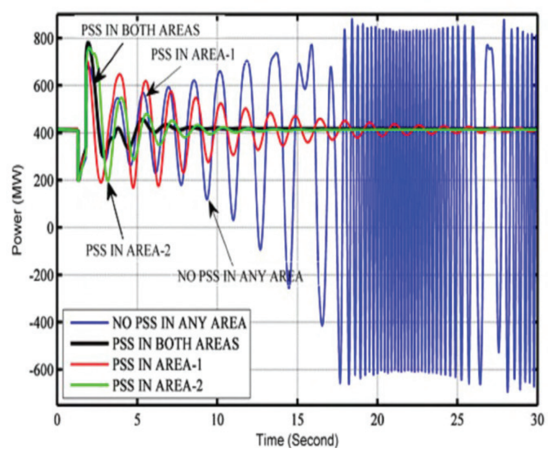


Figure 7.4: The power flow response from area 1 to area 2 of the 2 area test system to a fault at bus 8. Image from [1]

In the time domain simulation results from Parmar's original paper shown in Figures 7.4, 7.5, 7.6, and 7.7 we can clearly see that placing PSSs on generators 3 and 4 in area 2 produces the best damping of the systems response to a fault. Figure 7.4 confirms that without a PSS the system will become unstable as the response will not be naturally damped by the generators. The response of the system when the PSSs are placed at the generators in area 1 is damped similar to the area 2 placement

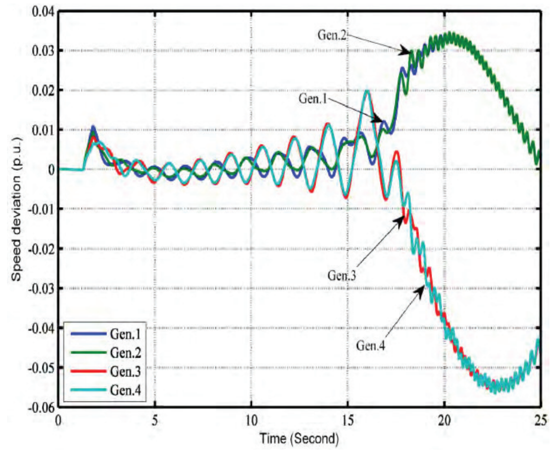


Figure 7.5: The speed deviation response of the 2 area test system generators to a fault at bus 8 with no PSSs. Image from [1]

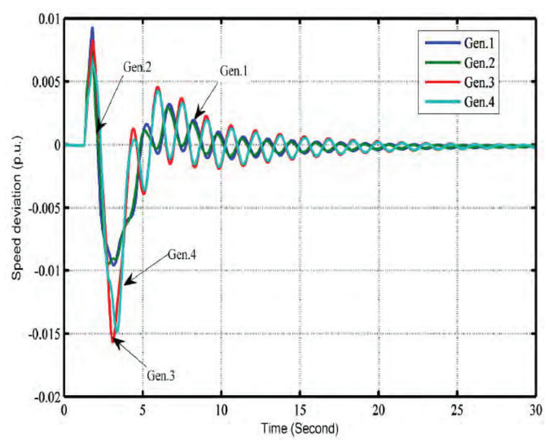


Figure 7.6: The speed deviation response of the 2 area test system generators to a fault at bus 8 with PSSs on the generators in area 1. Image from [1]

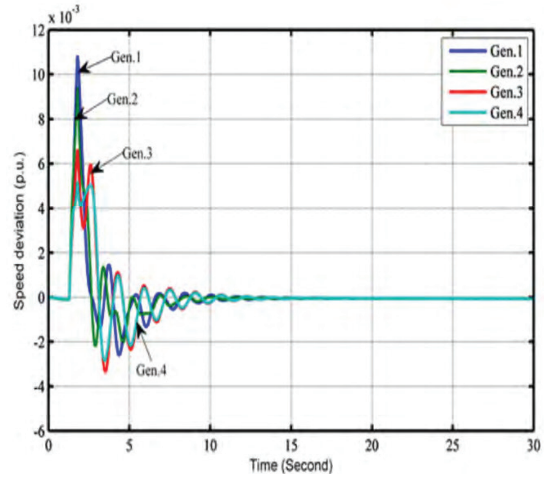


Figure 7.7: The speed deviation response of the 2 area test system generators to a fault at bus 8 with PSSs on the generators in area 2. Image from [1]

case. However, the damping occurs at a considerably slower rate. Therefore, for the 2 area test system, the state in mode participation factors provided the correct placement location while the mode in state participation factors failed to provide a clear suggestion.

7.4 9 Bus Results

7.4.1 Eigenanalysis

The results of the eigenanalysis for the less stable version of the 9 bus system used in this investigation are presented in Table 7.3. This system has a relatively small subset of modes that develop clear trajectories towards the imaginary axis as the load is increased. These are the modes at 1.78 Hz, 1.59 Hz, and 0.166 Hz. Of these modes, the 1.78 Hz one develops the fastest movement towards the imaginary axis. Though it is not near the imaginary axis at the start of the load increase, its

Table 7.3: Eigenvalue progression due to increased loading at bus 6. (The critical modes with clear trajectories are highlighted)

Eigenvalue Identifier	Loading	Real	Imaginary	Frequency
4	4.74	-0.90821	12.577	2.0069
	4.75	-0.90743	12.5735	2.0063
	4.76	-0.92104	12.576	2.0069
	Drift	-0.01283	-0.001	0
6		-0.24136	7.7115	1.2279
		-0.24173	7.7028	1.2265
		-0.24165	7.7809	1.239
	Drift	-0.00029	0.0694	0.0111
8		-8.0173	7.8138	1.7818
		-8.0729	7.8004	1.7866
		-7.4222	7.9276	1.7284
	Drift	0.5951	0.1138	-0.0534
10		-5.4757	7.9239	1.533
		-5.4781	7.9244	1.5332
		-5.4538	7.9193	1.5304
	Drift	0.0219	-0.0046	-0.0026
12		-6.0823	7.9755	1.5963
		-6.0889	7.9757	1.597
		-6.0046	7.9713	1.5883
	Drift	0.0777	-0.0042	-0.008
16		-0.46946	0.8568	0.15549
		-0.46952	0.85649	0.15545
		-0.46915	0.86036	0.15597
	Drift	0.00031	0.00356	0.00048
18		-0.85727	0.59014	0.16564
		-0.86228	0.58715	0.16603
		-0.79965	0.62072	0.16111
	Drift	0.05762	0.03058	-0.00453
20		-0.64586	0.77474	0.16053
		-0.64659	0.77428	0.16055
		-0.64243	0.78306	0.1612
	Drift	0.00343	0.00832	0.00067

rate of movement is great enough that it will cross the imaginary axis before the mode at 0.166 Hz. The mode at 0.166 Hz starts much closer to the imaginary axis and also moves towards the imaginary axis. However, its rate is much slower than the 1.78 Hz mode and so the 1.78 Hz mode appears to be more critical.

7.4.2 Participation Factor Analysis

The participation factor analysis for each of the modes of the 9 bus system can be found in Table 7.4. Considering the participation factors for the 1.78 Hz, 1.59 Hz, and 0.166 Hz modes we again see confusing and incorrect suggestions from the mode in state participation factors. We know from time domain simulation that placing the PSS on generator 1 produces the best damping of the system. However, the mode in state participation factors for the critical modes suggest placing the PSS on generators 2 and 3.

On the other hand, two of the state in mode participation factors for the critical modes suggest the correct placement, including the most critical mode (1.78 Hz). The state in mode participation factors for the other critical mode (0.166 Hz) suggest placing the PSS on generator 3. However, the participation factors for this mode are very close in value for all three generators unlike the other cases where the participation factor for one generator is clearly much greater than the others. Therefore, the 0.166 Hz mode in this system is much like the 0.96 Hz mode of the 2 area system which did not offer a clear suggestion on placement. Still, when the state in mode participation factors of the three critical modes are considered

Table 7.4: Participation Factor values for the synchronous machine related states for each mode of the system.

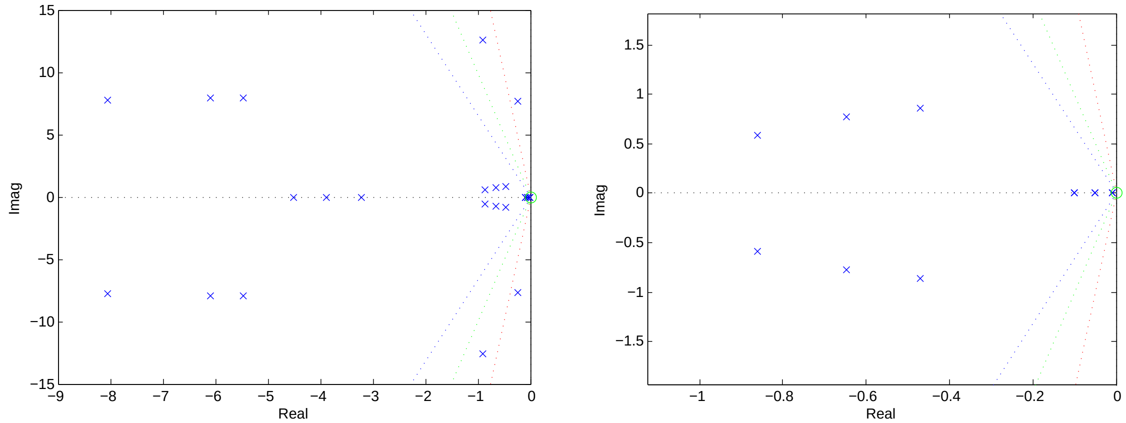
Mode in State			State in Mode		
Mode Frequency	PF Magnitude	Associated State	Mode Frequency	PF Magnitude	Associated State
0.155	0.4082	e1q_Syn_1	0.155	0.49982	delta_Syn_1
0.16	0.37776	e1q_Syn_2	0.16	0.49298	delta_Syn_1
0.166	0.40756	e1q_Syn_3	0.166	0.35976	delta_Syn_3
1.23	0.35877	delta_Syn_2, omega_Syn_2	1.23	0.49997	delta_Syn_1
1.53	0.01561	e1q_Syn_1	1.53	0.4917	delta_Syn_1
1.6	0.0167	e1q_Syn_2	1.6	0.43836	delta_Syn_1
1.79	1.26E-02	e1q_Syn_3	1.79	0.42727	delta_Syn_1
2	0.38914	delta_Syn_3, omega_Syn_3	2	0.49982	delta_Syn_3

together they clearly point to the correct placement of the PSS in the system.

In addition, the state in mode participation factors all suggest the rotor angle as the critical state variable while the mode in state participation factors all suggest the q-axis voltage of the rotor. In this case, the suggestion of rotor angle is sensible because the rotor angle is closely related to the rotor velocity which is a typical input for PSSs. The q-axis voltage suggestion, on the other hand, is less reasonable because PSSs do not typically use voltage as an input. However, PSAT does provide voltage as a potential input for the PSS.

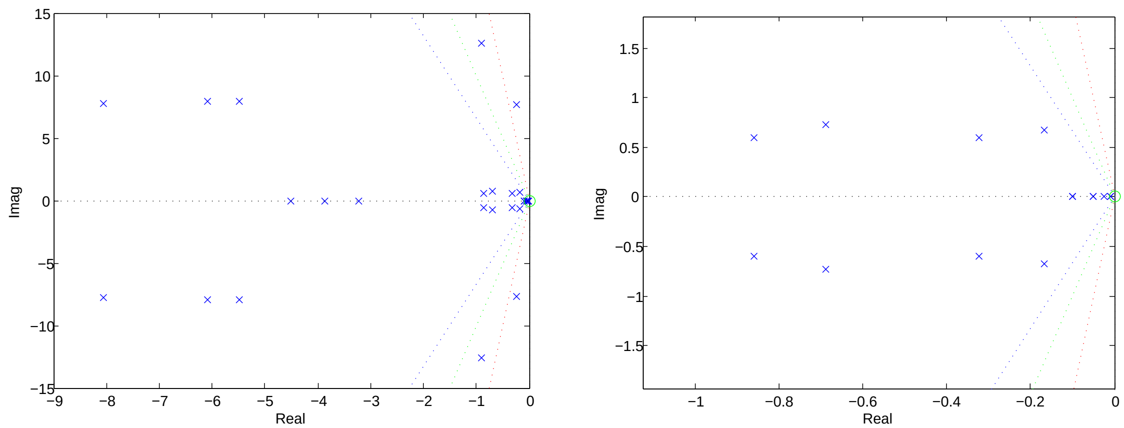
7.4.3 PSS Placement Eigenanalysis Results

The effect of PSS placement can be seen in the real components of the modes of a system. As the damping of troublesome modes increases, the real component of those modes also increases. Therefore, effective PSS placement should result in at least one mode moving farther away from the imaginary axis. This effect is evident in Figures 7.8, 7.9, 7.10, and 7.11 where the addition of a PSS clearly moves a mode away from the imaginary axis. A zoomed in view of the eigenvalues is provided since the mode that is moved away from the imaginary axis is very close to it initially. From this analysis it is clear that placing a PSS at generator 1 has the greatest effect as it moves the mode farther from the imaginary axis than the other placements.



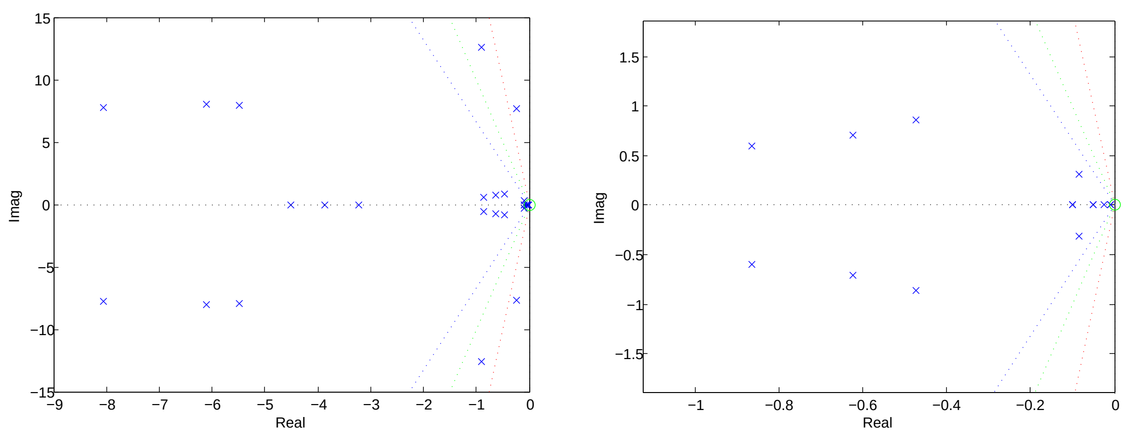
(a) Full View (b) Zoomed in View

Figure 7.8: The eigenvalues of the 9 Bus system without a PSS.



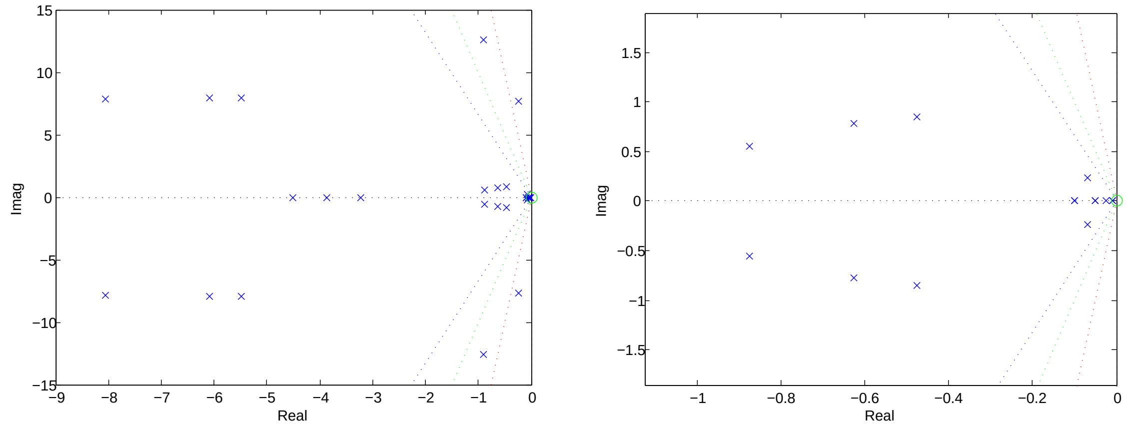
(a) Full View (b) Zoomed in View

Figure 7.9: The eigenvalues of the 9 Bus system with a PSS on generator 1.



(a) Full View (b) Zoomed in View

Figure 7.10: The eigenvalues of the 9 Bus system with a PSS on generator 2.



(a) Full View (b) Zoomed in View

Figure 7.11: The eigenvalues of the 9 Bus system with a PSS on generator 3.

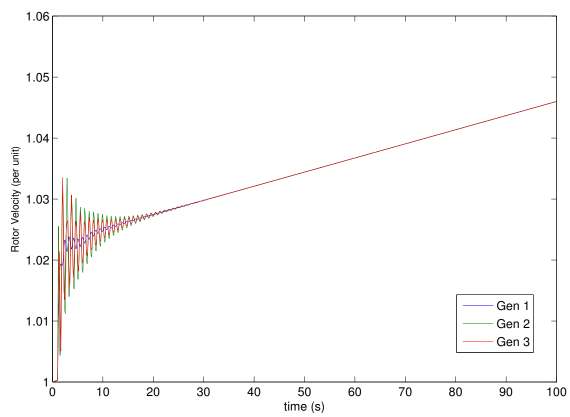


Figure 7.12: The rotor speed response of the 9 bus system generators to a fault at bus 7 with no PSSs.

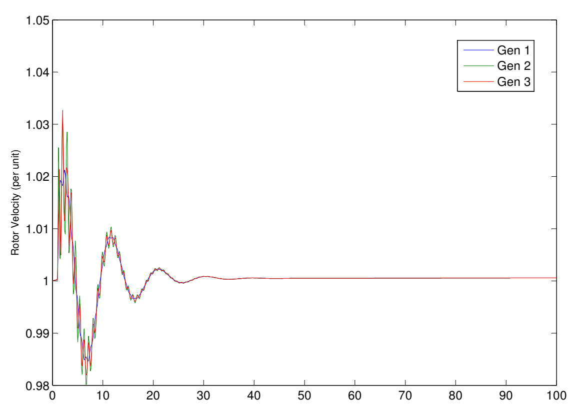


Figure 7.13: The rotor speed response of the 9 bus system generators to a fault at bus 7 with a PSS on generator 1 using rotor speed as its input.

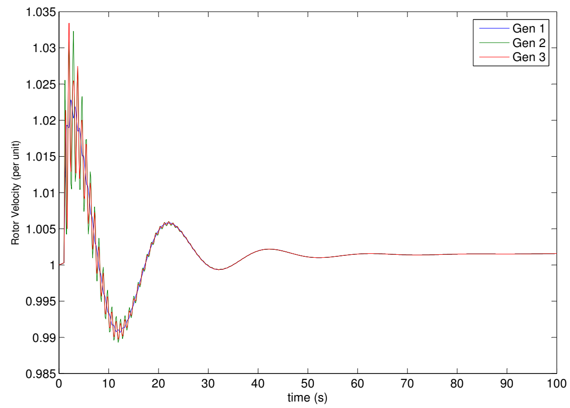


Figure 7.14: The rotor speed response of the 9 bus system generators to a fault at bus 7 with a PSS on generator 2 using rotor speed as its input.

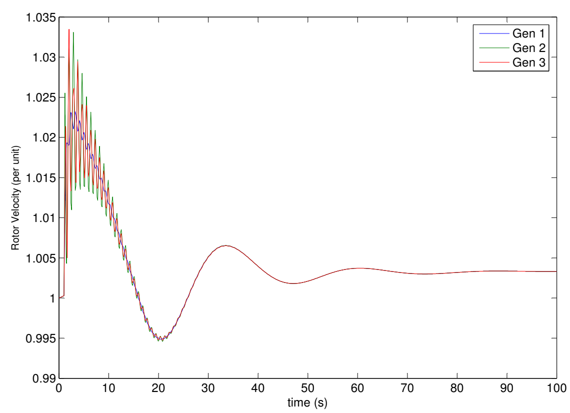


Figure 7.15: The rotor speed response of the 9 bus system generators to a fault at bus 7 with a PSS on generator 3 using rotor speed as its input.

7.4.4 PSS Placement Time Domain Results

Figures 7.12, 7.13, 7.14, and 7.15 show the time domain simulations of the system response to a 200ms three phase fault at bus 7 for the different PSS placements. When there is no PSS included in the system, the rotor velocity continuously increases even after the oscillations have been damped. The addition of a PSS solves this problem but introduces some larger oscillations which damp slowly. Placing the PSS at generator 1 provides the most rapid damping of these larger oscillations. In addition, the PSS placement at generator 1 also allowed the system to return to a steady-state operating point that was nearest to its initial operating point. Therefore, the placement consistently suggested by the state in mode participation factor is the optimal placement for this system.

7.4.5 Input Alternatives

As was discussed in Chapter 6, there are several options for the input to the power system stabilizer. PSAT provides three input options for the PSSs it simulates: velocity, power and voltage. In the time domain simulations above the PSS was always using the velocity input. For the sake of completeness we have also included the results for the other two kinds of input available in Figures 7.16 and 7.17. With the current PSS tuning parameters these inputs produce an undesired drop in generator velocity. It may be possible to correct this behavior with an adjustment to the tuning of the PSS however all attempts to improve the response have had little effect. More advanced tuning algorithms appear to be required and that is beyond the scope of this work.

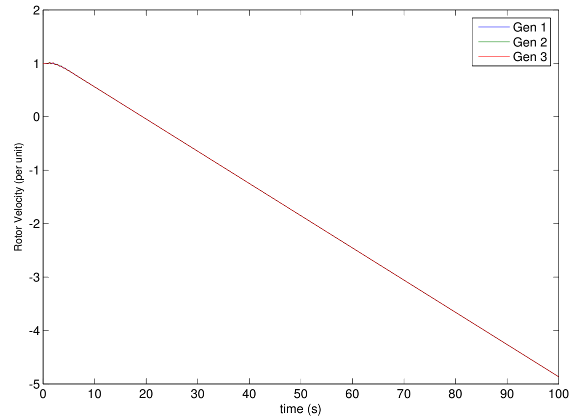


Figure 7.16: The rotor speed response of the 9 bus system generators to a fault at bus 7 with a PSS on generator 1 using power as its input.

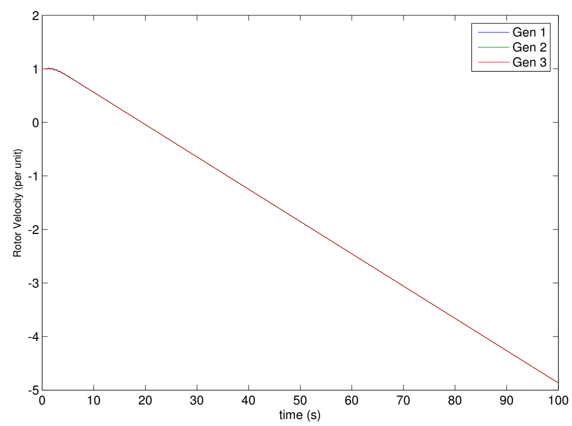


Figure 7.17: The rotor speed response of the 9 bus system generators to a fault at bus 7 with a PSS on generator 1 using voltage as its input.

7.5 Discussion

The results from the example systems presented in this chapter featured several interesting similarities. First, both of the systems had a subset of modes which developed a clear trajectory towards the imaginary axis. Second, both of them had one mode whos participation factors did not provide a clear indication of where to place the PSS necessitating the use of multiple modes to generate a clear suggested placement. Third, in both cases, the mode in state participation factors suggested the wrong placement while the state in mode participation factors suggested the correct placement. All of these factors combined seem to indicate some correlation between the state in mode participation factors of critical modes in a system and the correct PSS placement for that system. Therefore, these results provide some numerical support for the hypothesis that the state in mode participation factors are useful for providing information that can help with control placement applications.

However, it is important to note that we have been unable to identify a direct connection between participation factor analysis and control siting from a mathematical stand point. These numerical results seem to indicate that there is some sort of indirect relationship though. It could be that the state in mode participation factors reveal which states are most sensitive to perturbation. These states would then react more strongly to the addition of controls and this behavior could be responsible for the results presented in this chapter. However, a location's sensitivity to the addition of controls does not necessarily indicate that that location represents an optimal placement. So while state in mode participation factors may be a useful tool to identify promising locations, other methods are probably needed to guarantee optimal control placement.

In addition, because the state in mode participation factor definition is different from the original participation factor definition, this may explain why sometimes the participation factor analysis did not agree with the optimal placement in previous work. For some systems, the optimal locations suggested by both kinds of participation factors will agree and the placement will work out. However, for some other systems the state in mode participation factor might provide a different location than the mode in state participation factor. In those systems the original participation factor definition (the current mode in state participation factor definition) would have suggested a sub-optimal placement leading to the observed disagreement between participation factor analysis and best placement location.

Chapter 8: Conclusions

The results presented in this thesis provide numerical evidence that supports the hypothesis that the intuitive applications of mode in state and state in mode participation factors are in fact the correct applications. Mode in state participation factors, which gauge the effect that each system mode has on each system state, are best used for monitoring applications. This is intuitive because the state which is most effected by a particular mode is the obvious choice for measurement to observe the mode in question. State in mode participation factors, on the other hand, are best used to provide some useful information for control applications. This is because they assess the effect that each state has on each mode. If you want to adjust some mode of the system, it makes sense to adjust the state variable that has the greatest affect on that mode. However, despite the numerical support for a connection between state in mode participation factors and control applications, there is currently no clear direct mathematical relationship between the two.

Another interesting aspect of the results presented in this thesis is that the use of the two kinds of participation factors in their appropriate applications sometimes yields unintuitive yet effective suggestions. This would seem to indicate that these participation factors are revealing some additional information about the systems.

In addition, the voltage stability analysis results presented a novel online proximity to instability evaluation method. This scheme utilizes participation factors to identify which critical state variables to analyze with Prony analysis. The real world equivalents of these state variables can then be analyzed with Prony analysis to determine the modes of the system as was suggested in [9]. This modal information can then be used to establish a relative measure of the system's proximity to voltage instability similar to the method in [3]. To the best of our knowledge, this is the first time that participation factors have been suggested as a way to improve the accuracy of Prony analysis.

Furthermore, the power system stabilizer placement results provide some explanation for the inconsistent effectiveness of participation factors for this application in the past. The previous definition of participation factors actually corresponded only to the mode in state participations. However, the state in mode participation factors are hypothetically better suited to control placement applications. Therefore, the original participation factors potentially provided suggestion for PSS placement locations that were less than optimal. In some systems, the two different participation factor definitions would produce the same suggestion. Those systems seemed to support the notion that participation factors provided good placement suggestions. For other systems, though, the two kinds of participation factors produced different suggestions. In those cases, the original definition produced suggestions that were sub-optimal. This made it seem though participation factors only worked for placement applications some of the time. In reality, the problem was that the wrong definition was being used and that the correct state in mode definition would have provided the optimal placement.

8.1 Future Work

In the future, the conclusions of this thesis should be tested using larger more complex systems to verify that they hold up for more realistic systems. One of Prony's advantages is its ability to extract correct modal information from a signal in the presence of noise. Another area of future work would be to verify these results in systems where noise is present. In addition, while this thesis provides some numerical evidence supporting the appropriate applications of the different types of participation factors, mathematical proofs explaining and supporting these observations are needed and should be developed. Finally, the state in mode participation factors should be explored further as a way to identify system state variables that are particularly sensitive to perturbations. Understanding this relationship between state in mode participation factors and sensitivity to perturbations may also help to explain why state in mode participation factors seems to provide better control placement suggestions than mode in state participation factors.

Bibliography

[1] D. Parmar, V. Dholakiya, and S. Vora, "Optimal placement of power system stabilizers: Simulation studies on a test system," in *Engineering (NUiCONE), 2011 Nirma University International Conference on*, 2011, pp. 1–6.

[2] I. Dobson, H. Glavitsch, C.-C. Liu, Y. Tamura, and K. Vu, "Voltage collapse in power systems," *Circuits and Devices Magazine, IEEE*, vol. 8, no. 3, pp. 40–45, 1992.

[3] C. Sharma and M. Ganness, "Determination of power system voltage stability using modal analysis," in *Power Engineering, Energy and Electrical Drives, 2007. POWERENG 2007. International Conference on*, 2007, pp. 381–387.

[4] D. Aik and G. Andersson, "Use of participation factors in modal voltage stability analysis of multi-infeed hvdc systems," *Power Delivery, IEEE Transactions on*, vol. 13, no. 1, pp. 203–211, 1998.

[5] Y. Mansour, W. Xu, F. Alvarado, and C. Rinzin, "Svc placement using critical modes of voltage instability," *Power Systems, IEEE Transactions on*, vol. 9, no. 2, pp. 757–763, 1994.

[6] B. Gao, G. K. Morison, and P. Kundur, "Voltage stability evaluation using modal analysis," *Power Systems, IEEE Transactions on*, vol. 7, no. 4, pp. 1529–1542, 1992.

[7] F. Milano, "An open source power system analysis toolbox," *Power Systems, IEEE Transactions on*, vol. 20, no. 3, pp. 1199–1206, 2005.

[8] J. Hauer, C. Demeure, and L. Scharf, "Initial results in prony analysis of power system response signals," *Power Systems, IEEE Transactions on*, vol. 5, no. 1, pp. 80–89, 1990.

[9] J. Hauer, "Application of prony analysis to the determination of modal content and equivalent models for measured power system response," *Power Systems, IEEE Transactions on*, vol. 6, no. 3, pp. 1062–1068, 1991.

[10] L. Qi, L. Qian, S. Woodruff, and D. Cartes, "Prony analysis for power system transient harmonics," *EURASIP J. Appl. Signal Process.*, vol. 2007, no. 1, pp. 170–170, Jan. 2007. [Online]. Available: http://dx.doi.org/10.1155/2007/48406

[11] I. Perez-Arriaga, G. Verghese, and F. Schweppe, "Selective modal analysis with applications to electric power systems, part i: Heuristic introduction," *Power Apparatus and Systems, IEEE Transactions on*, vol. PAS-101, no. 9, pp. 3117–3125, 1982.

[12] G. Verghese, I. Perez-Arriaga, and F. Schweppe, "Selective modal analysis with applications to electric power systems, part ii: The dynamic stability problem," *Power Apparatus and Systems, IEEE Transactions on*, vol. PAS-101, no. 9, pp. 3126–3134, 1982.

[13] A. Mendonca and J. Peas Lopes, "Robust tuning of pss in power systems with different operating conditions," in *Power Tech Conference Proceedings, 2003 IEEE Bologna*, vol. 1, 2003, pp. 7 pp. Vol.1–.

[14] Y.-Y. Hsu and C.-L. Chen, "Identification of optimum location for stabiliser applications using participation factors," *Generation, Transmission and Distribution, IEE Proceedings C*, vol. 134, no. 3, pp. 238–244, 1987.

[15] W. Hashlamoun, M. Hassouneh, and E. Abed, "New results on modal participation factors: Revealing a previously unknown dichotomy," *Automatic Control, IEEE Transactions on*, vol. 54, no. 7, pp. 1439–1449, 2009.

[16] P. Kundur, *Power system stability and control.* McGraw-Hill, 2001, vol. 12.

[17] P. Kundur, J. Paserba, V. Ajjarapu, G. Andersson, A. Bose, C. Canizares, N. Hatziargyriou, D. Hill, A. Stankovic, C. Taylor, T. Van Cutsem, and V. Vittal, "Definition and classification of power system stability ieee/cigre joint task force on stability terms and definitions," *Power Systems, IEEE Transactions on*, vol. 19, no. 3, pp. 1387–1401, 2004.

[18] B. Gao, G. K. Morison, and P. Kundur, "Towards the development of a systematic approach for voltage stability assessment of large-scale power systems," *Power Systems, IEEE Transactions on*, vol. 11, no. 3, pp. 1314–1324, 1996.

[19] G. Andersson, P. Donalek, R. Farmer, N. Hatziargyriou, I. Kamwa, P. Kundur, N. Martins, J. Paserba, P. Pourbeik, J. Sanchez-Gasca, R. Schulz, A. Stankovic, C. Taylor, and V. Vittal, "Causes of the 2003 major grid blackouts in north america and europe, and recommended means to improve system dynamic performance," *Power Systems, IEEE Transactions on*, vol. 20, no. 4, pp. 1922–1928, 2005.

[20] V. Balamourougan, T. Sidhu, and M. Sachdev, "Technique for online prediction of voltage collapse," *Generation, Transmission and Distribution, IEE Proceedings-*, vol. 151, no. 4, pp. 453–460, 2004.

[21] S. Greene, I. Dobson, and F. Alvarado, "Contingency ranking for voltage collapse via sensitivities from a single nose curve," *Power Systems, IEEE Transactions on*, vol. 14, no. 1, pp. 232–240, 1999.

[22] P.-A. Lof, T. Smed, G. Andersson, and D. Hill, "Fast calculation of a voltage stability index," *Power Systems, IEEE Transactions on*, vol. 7, no. 1, pp. 54–64, 1992.

[23] C. Grund, J. J. Paserba, J. Hauer, and S. Nilsson, "Comparison of prony and eigenanalysis for power system control design," *Power Systems, IEEE Transactions on*, vol. 8, no. 3, pp. 964–971, 1993.

[24] M. Johnson, I. P. Zarafonitis, and M. Calligaris, "Prony analysis and power system stability-some recent theoretical and applications research," in *Power Engineering Society Summer Meeting, 2000. IEEE*, vol. 3, 2000, pp. 1918–1923 vol. 3.

[25] S. Kay and J. Marple, S.L., "Spectrum analysis; a modern perspective," *Proceedings of the IEEE*, vol. 69, no. 11, pp. 1380–1419, 1981.

[26] S. Chakrabarti, E. Kyriakides, T. Bi, D. Cai, and V. Terzija, "Measurements get together," *Power and Energy Magazine, IEEE*, vol. 7, no. 1, pp. 41–49, 2009.

[27] S. Marple, *Digital spectral analysis with applications.* Prentice-HallEnglewood Cliffs, NJ, 1987.

[28] S. Singh, "Application of prony analysis to characterize pulsed corona reactor measurements," Master's thesis, The University of Wyoming, 2003.

[29] ——. (2003) Prony toolbox. [Online]. Available: http://www.mathworks.com/matlabcentral/fileexchange/3955

[30] M. Basler and R. Schaefer, "Understanding power system stability," in *Protective Relay Engineers, 2005 58th Annual Conference for*, 2005, pp. 46–67.

[31] A. Feliachi, "Optimal siting of power system stabilisers," *Generation, Transmission and Distribution, IEE Proceedings C*, vol. 137, no. 2, pp. 101–106, 1990.

[32] V. Arcidiacono, E. Ferrari, R. Marconato, J. Ghali, and D. Grandez, "Evaluation and improvement of electromechanical oscillation damping by means of eigenvalue-eigenvector analysis. practical results in the central peru power system," *Power Apparatus and Systems, IEEE Transactions on*, vol. PAS-99, no. 2, pp. 769–778, 1980.

[33] F. de Mello, P. J. Nolan, T. Laskowski, and J. Undrill, "Coordinated application of stabilizers in multimachine power systems," *Power Apparatus and Systems, IEEE Transactions on*, vol. PAS-99, no. 3, pp. 892–901, 1980.

[34] T. Hiyama, "Coherency-based identification of optimum site for stabiliser applications," *Generation, Transmission and Distribution, IEE Proceedings C*, vol. 130, no. 2, pp. 71–74, 1983.

[35] Z. Wang, C. Chung, K. Wong, D. Gan, and Y. Xue, "Probabilistic power system stabilizer design with consideration of optimal siting using recursive genetic algorithm," *European Transactions on Electrical Power*, vol. 21, no. 3, pp. 1409–1424, 2011.